Distributed coordination of fractionnal-order multi-agent systems

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À mes parents,

à toute ma famille,

à mes professeurs,

et à mes chèr(e)s ami(e)s.

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Villeneuve d'Ascq, France *Jing BAI*
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Introduction

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1.1 Background and Motivation

Along with the development of modern control technology, the multi-agent systems have been widely studied in recent years. Before presenting our main research results, the background and motivation on distributed coordination of fractional-order multi-agent systems are introduced by the following two parts: multi-agent systems and state of the art. In the first part, some definitions, applications on multi-agent systems from network topology, distributed coordination and fractional-order multi-agent systems aspects will be given. In the second part, based on our main research, we focus on stating recent results on consensus, formation control and fractional-order systems. At the same time, the study motivation will be proposed based on the background.

1.1.1 Multi-agent systems

A multi-agent system is a computerized system composed of multiple interacting intelligent agents within an environment. In recent years, multi-agent systems have been widely researched in biology, physics, apply mathematics, mechanics and control theory. The applications of multi-agent systems are diverse (Murray 2007; Peng *et al.* 2013b; Reynolds 1987; Štula *et al.* 2013), ranging from the motion of a flock of birds, a herd of land animals, a school of fishes and a swarming of bacterias in natural systems (see Fig. 1.1), to multiple air vehicles, multiple underwater vehicles, multiple mobile robots, multiple satellites in man-made systems (see Fig. 1.2). Compared to a single agent system, multi-agent systems are capable of executing more complex tasks due to their great advantages, such as improving system efficiency, flexibility and reliability, reducing cost, and providing new capability. In study, the agents communication relation can be described by the following network topology.

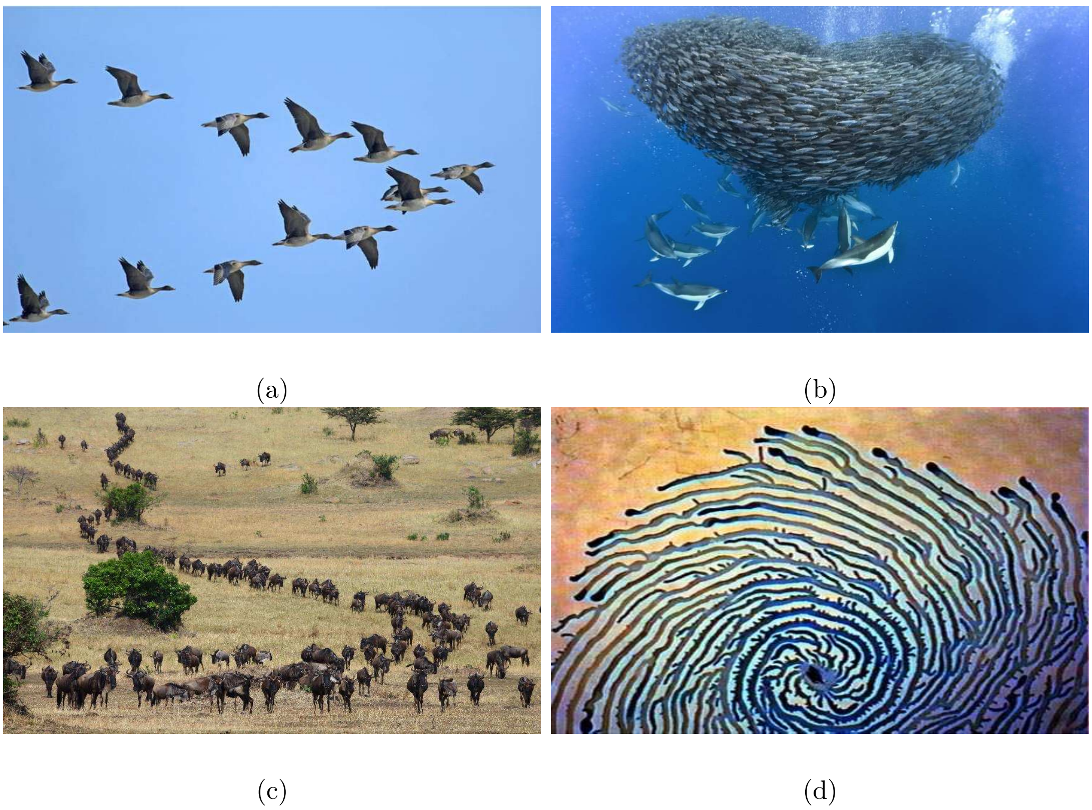
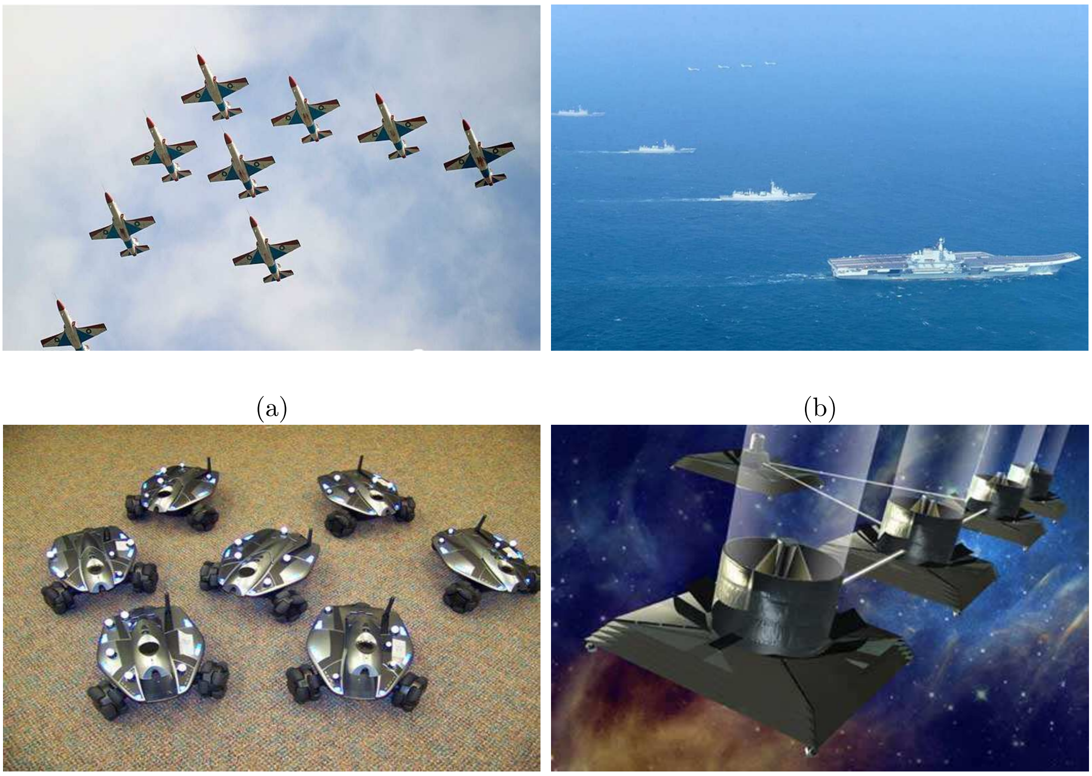


Figure 1.1: Examples of multi-agent systems in natural (a) flocks of birds; (b) school of fishes; (c) herd of horses; (d) swarming of bacterias.



(a) (b)

(c) (d)

Figure 1.2: Examples of multi-agent systems in man-made (a) air vehicles; (b) underwater vehicles; (c) multiple mobile robots; (d) multiple satellites.

1.1.1.1 Network topology

As defined in the above section, multi-agent system includes multiple interacting intelligent agents hence the system can be described as a communication network. Its topology is usually a schematic description of the arrangement of the communication network, where the nodes represent agents and the lines are information exchanged between agents. In our study, the topology can be divided into four types: fixed topology, switching topology, undirected topology and directed topology. We will introduce them from their definitions and the corresponding graph theories.

Fixed topology: if the communication network among agents is fixed no change all time, its topology is called fixed topology.

It becomes a challenge when the agents communication relation is changing with time (Qin *et al.* 2011; Zhou & Wang 2009), for example, one agent cut its information with others or change its communication neighbors and so on. In these cases, the switching topology is used.

Switching topology: if the communication networks among agents is time variable, the topology is called switching topology.

According to the information transmission among agents, undirected topology and directed topology are used.

Undirected topology: if agent i can receive information from agent j, and agent j can receive information from agent i, the topology is called undirected topology.

Directed topology: if the topology is not an undirected topology, is called a directed topology.

Remark 1.1 *For simplicity of presentation, we suppose that all agents work in a one-dimensional space, and all results hereafter are still valid for the m-dimensional* $(m > 1)$ *case by introduction of Kronecker product.*

For the above topologies, which can be described by the corresponding graphs: fixed graph, switching graph, undirected graph and directed graph.

A system consisting of n agents, the interaction fixed graph for all agents can be modeled as follows. Let $G = \{V, E\}$ be a fixed weighted communication graph, where $V = \{v_1, v_2, \cdots, v_n\}$ represents a finite nonempty set nodes, and $E \subseteq V \times V$ is a set of ordered pairs of nodes, called edges. Each edge can be denoted as $e_{i,j} = (v_i, v_j)$.

Fixed graph: If the graph $G = \{V, E\}$ is fixed no change all time, the graph is a fixed graph (see Fig. 1.3).

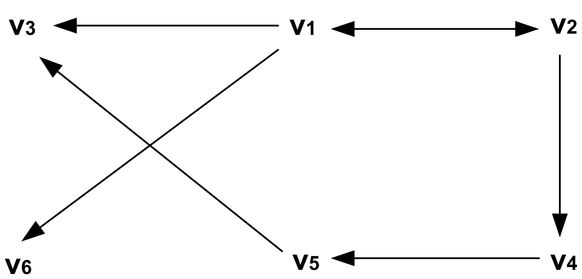


Figure 1.3: A fixed directed graph with six agents.

Switching graph: Let $G_{\sigma(t)} = \{V, E\}$ describe a graph, where $\sigma(t)$ is a switching signal defined as $\sigma(t) : [0, \infty) \Rightarrow \{1, p\}$ which is a piecewise constant function, p denotes the total number of all possible communication graph. Suppose that the graph switches only at time $t_i, i = 0, 1, \cdots$ and $t_0 = 0s$, in each time interval the communication graph is fixed.

To show the switching graph clearly, an example is shown as Fig. 1.4, where $p = 2$ and the communication relation changes at time t_1.

Figure 1.4: A switching graph with six agents.

In reality, the undirected graph and directed graph are often used, we give them in the case of fixed graph.

Undirected graph: $G = \{V, E\}$ is defined as undirected graph if for all $v_i, v_j \in V$,

$$(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E \tag{1.1}$$

The pairs of nodes in an undirected graph are unordered, an example of undirected graph with six agents is shown in Fig. 1.5.

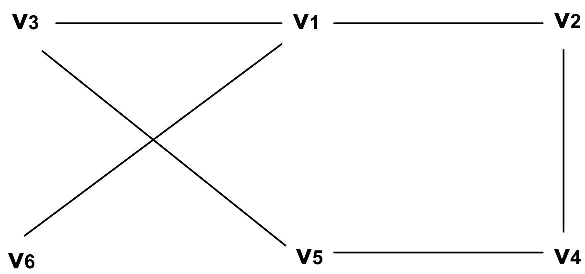


Figure 1.5: An undirected graph with six agents.

Directed graph: $G = \{V, E\}$ is defined as directed graph if it is not a undirected.

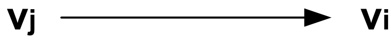


Figure 1.6: Information flow from agent j to agent i.

In directed graph (see Fig. 1.3), we refer to v_i and v_j as the tail and the head of the edge (v_i, v_j) as follows

Directed path is a sequence of edges in a directed graph with the form $(v_1, v_2), (v_2, v_3), \cdots$, where $v_i \in V$. Then, we can give the definition of directed spanning tree.

Directed spanning tree: if at least one node in graph $G = \{V, E\}$ has a directed path to all other nodes, then, the directed path is a directed spanning tree.

A fixed directed graph G of n agents can be represented by the weighted adjacency matrix A and the Laplace matrix L.

Definition 1.2 *The weighted adjacency matrix A of directed communication graph G is defined as*

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{pmatrix} \in R^{n\times n} \tag{1.2}$$

where $a_{i,j}$ is the weight of edge (v_j, v_i), which describes the communication quality between agent i and agent j, and

$$\begin{cases} a_{i,j} > 0, & (v_j, v_i) \in E \\ a_{i,j} = 0, & otherwise \end{cases} \tag{1.3}$$

Remark 1.3 *For the numerical simulations, we suppose that the weight $a_{i,j} = 1$ when $(v_j, v_i) \in E$ and otherwise $a_{i,j} = 0$.*

Definition 1.4 *The Laplace matrix $L = (l_{ij})_{n\times n}$ of a directed communication* *graph is defined as*

$$l_{ij} = \begin{cases} \sum_{j \in N_i} a_{i,j}, & i = j \\ -a_{i,j}, & (v_j, v_i) \in E \text{ and } i \neq j \\ 0, & otherwise \end{cases} \tag{1.4}$$

L can be given by

$$L = \begin{pmatrix} \sum_{j \in N_1} a_{1,j} & -a_{1,2} & \cdots & -a_{1,n} \\ -a_{2,1} & \sum_{j \in N_2} a_{2,j} & \cdots & -a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n,1} & -a_{n,2} & \cdots & \sum_{j \in N_n} a_{n,j} \end{pmatrix}. \tag{1.5}$$

The following equation can be easily obtained

$$L\mathbf{1} = 0, \tag{1.6}$$

where vector $\mathbf{1} = (1, 1, \cdots, 1)^T$.

Example 1.5 *Consider the fixed directed graph Fig. 1.3. Then, its weighted adjacency matrix and the Laplace matrix can be given as follows*

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1.7}$$

Based on the above graph theories, the following lemma can be given, which will play an important role in the stability analysis.

Lemma 1.6 *(Ren 2007; Shen et al. 2012) For a fixed communication graph G, $x = [x_1, \cdots, x_n]^T$, $x_i \in R$, then the following conditions are equivalent:*
(1) The communication graph G has a directed spanning tree;
(2) $L \in R^n$ has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}$ and other eigenvalues have positive real parts, where $\mathbf{1} = (1, 1, \cdots, 1)^T$;
(3) $Lx = 0$ implies that $x_1 = \cdots = x_n$.

In the switching case, the weighted adjacency matrix $A(t)$ is time variable, its element can be chosen as the following form

$$a_{i,j}(t) = \begin{cases} a_{i,j}, & (v_j, v_i) \in E \\ 0. & otherwise \end{cases} \tag{1.8}$$

In this thesis, we consider the fixed directed communication graph, the switching one will be our future work.

1.1.2 Distributed coordination

Due to the advantages of multi-agent systems, its control has received increasing demands. Two approaches are commonly used for controlling multi-agent systems: a centralized control and a distributed coordination control.

The centralized control approach assumes that a powerful decision maker is more effective to dominate other agents for achieving the tasks. This approach is represented by a simple schematic diagram in Fig. 1.7, where the robots represent the agents, the lines are the information transformation. The centralized controller sends to the agents specific commands. There are many practical applications using this method, for example, a centralized approach was used to create a RSBK (Robust Safe But Knowledgeable) control law applied to trajectory optimization problem of unmanned vehicles (Bellingham *et al.* 2002). In another application, a centralized approach was proposed to minimize the global quantity of potential conflicts in traffic dynamics, due to the fact that the central controller can quickly send advice to equipped vehicles (Monteil & Billot 2011). Also, the centralized approach was used to control multiple robots (Carelli *et al.* 2006; Mas & Kitts 2010), but this approach is impractical when systems include a large number of agents.

The distributed coordination control approach does not require a central maker for controlling systems. In distributed coordination of multi-agent systems, the main objective is to have the whole group of agents working in a cooperative fashion through decentralized controllers with local information and limited inter-agent communication, a simple schematic diagram Fig. 1.8 is given to show the process of distributed control. Here, information sharing plays a central role for achieving the cooperation objective. In addition, the distributed coordination has many advantages, especially low operational costs, less system

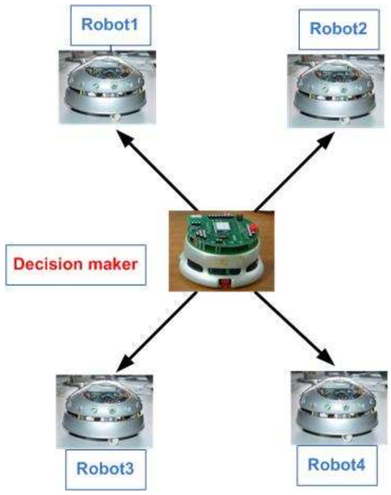


Figure 1.7: The centralized control.

requirement, high robustness, more adaptive, and flexible scalability (Cao *et al.* 2013). Hence, distributed coordination control of multi-agent systems is adopted for a broad range of control applications, including rendezvous (Dimarogonas & Kyriakopoulos 2007; Smith *et al.* 2007), sensor networks (Olfati-Saber 2007), robotic teams (Guruprasad & Ghose 2013; Lin *et al.* 2012), satellites formation (Wu *et al.* 2010), flocking (Yang *et al.* 2012), complex networks (Lu *et al.* 2011) and so on.

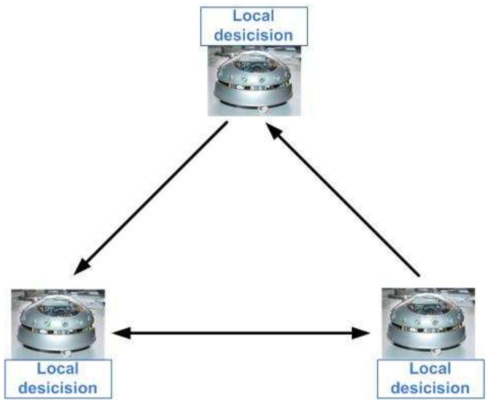


Figure 1.8: The distributed coordination control.

The recent researches on distributed coordination mainly include the following directions (Cao *et al.* 2013), **consensus** (Diao *et al.* 2014; Wen *et al.* 2013b), **formation control** (Zhao *et al.* 2014), **task assignment** (Shao *et al.* 2007) and **estimation** (Choi & Horowitz 2010; Cortes 2010). The following subsections will introduce this four directions from their definitions, applications and state of the arts.

1.1.2.1 Consensus

Consensus plays an important role in distributed coordination, this means that a group of agents would reach an agreement on a parameter, e.g., position, velocity, phase or attitude by interacting with their neighboring agents. Consensus has received considerable attention due to its broad application in cooperative control of air vehicles (Murray 2007) and underwater swarm robots (Joordens & Jamshidi 2010), flocking of mobile agents (Yang *et al.* 2010), cluster satellites (Liu *et al.* 2011) (see Fig. 1.9)and so on.

Consensus has also been considered as part of other distributed coordination problems, e.g.: in flocking, all agents move together with the same velocity (velocity consensus). In formation control, all agents maintain their relative position to one another (consensus on relative position) to form a desired formation shapes. In our study, consensus can be divided into two problems: consensus producing and consensus tracking. To definite these two cases, we first introduce a reference state (leader).

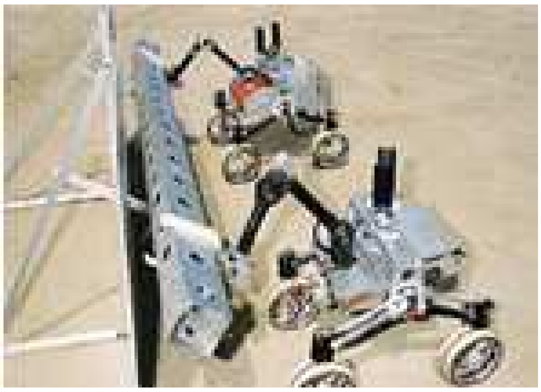
Reference state: a reference state represents a control objective or a common interest of the whole multi-agent group, a reference state is also called a leader.

Consensus producing: if multi-agents are not required to track a reference state, the consensus problem is called consensus producing.

Consensus tracking: if multi-agents are required to track a reference state, the consensus problem of multi-agent systems is called consensus tracking.

For the consensus producing problem, the final consensus value is inherent and decided by agents. A simple example with three agents is shown in Fig. 1.10, where the final consensus value is a constant, the agents reach the same point eventually. However, in many practical applications, it is desirable that the states of all agents can asymptotically track a reference state. The following example Fig. 1.11 shows the tracking process, where three agents track a reference state (a solid line). Examples of real applications include formation flying, body guard. Therefore, it is necessary to investigate a control law which can conduct agents to a reference state (Cao *et al.* 2009; Guoguang Wen & Yu 2011; Hong *et al.* 2006;

(a)



(b)

(c)

(d)

Figure 1.9: Examples of consensus (a) air vehicles; (b) underwater swarm robots; (c) flocking of mobile agents; (d) cluster satellites.

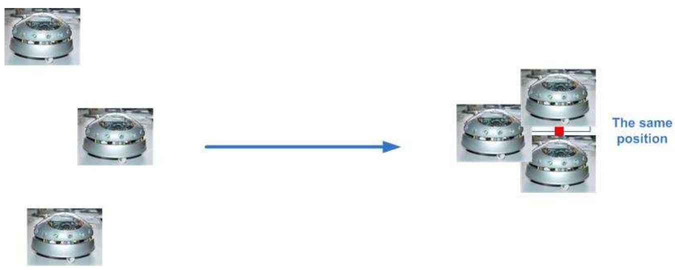


Figure 1.10: The problem of consensus producing.

Ji *et al.* 2008; Peng & Yang 2009; Ren 2007). Here a reference state can be a constant target or a time-varying state.

The consensus study has a long process, as above discussion, consensus can be viewed as a part of other distributed coordination, next we focus on introducing its **state of the art**.

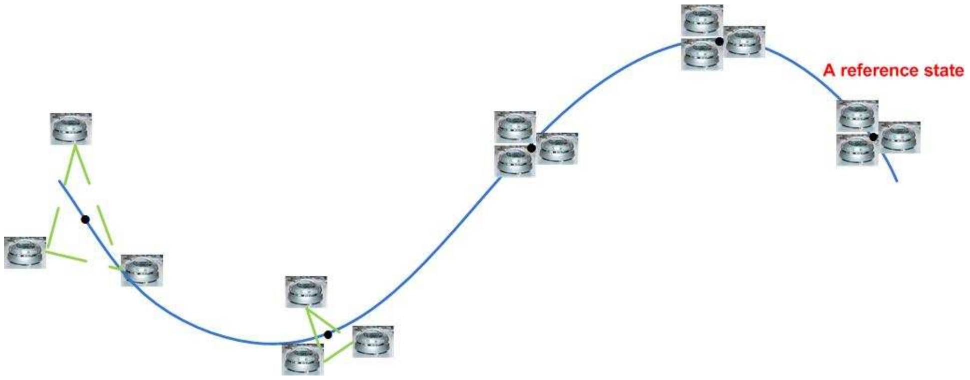


Figure 1.11: The problem of consensus tracking.

The study of consensus problem has a long history in multi-agent control. For example, all agents eventually move in the same direction without centralized coordination in (Vicsek *et al.* 1995), and a theoretical explanation for the behavior (Jadbabaie *et al.* 2003) was provided using graph theory. Moreover, various consensus algorithms were designed depending on different techniques, such as adaptive control approach (Dong 2012), sliding mode control method (Rao & Ghose 2011), and distributed pinning control method (Wen *et al.* 2013a). Recently, Due to the outside disturbance and physical limitation in practical systems, researches on consensus producing/tracking focus on time delay, convergence speed, finite time convergence, sample-data setting and so on.

For the study of **time delay**, the main problem is whether consensus can be achieved ultimately when time delay exists. For instance, Sun *et al.* 2008 considered average consensus under undirected networks of dynamics agents with fixed and switching topologies as well as multiple communication delay. The maximum allowed time delay (Olfati-Saber & Murray 2004) was obtained to judge if consensus is damaged in continuous time system. Different from above studies using matrix theory, the effect of time delay was considered based on the frequency domain analysis in (Tian & Liu 2008). In addition, time delay were investigated under more complex dynamics, such as second dynamics (Qin *et al.*

2011), complex networks (Wang *et al.* 2010b) and nonlinear dynamics (Hu *et al.* 2015).

The study of **convergence speed** focuses on how fast consensus is reached. In order to increase the convergence speed, the relative corresponding optimization approach is used (Xiao *et al.* 2009). Moreover, others authors (Zhou & Wang 2009) considered the measurement of convergence speed. The study of finite time convergence can be viewed as an extension of the study of convergence speed, which focuses on designing a controller such that state consensus among agents can be achieved when $t \geq T$, where T is a constant, which is also called consensus time.

Finite time convergence for single-integrator dynamics and double-integrator dynamics were proposed in (Li *et al.* 2011; Sayyaadi & Doostmohammadian 2011) respectively, the finite time convergence was also investigated in the discrete-time dynamics (Yuan *et al.* 2013).

The study of **sample-data framework** is used to handle the limitation inherits in of physical measurements and control input, which are described in a piece wise constant fashion. The main research question is to obtain conditions on the sampling period, which can be constant or time-varying. Consensus of multi-agent systems under single-integrator and double-integrator (Xie *et al.* 2009; Zhang & Tian 2010) were considered under sample-data framework respectively.

At the same time, various approaches were adopted, which include matrix theory (Zhang & Tian 2010), Lyapunov functions (Gao *et al.* 2013), stochastic matrices (Cao & Ren 2010b), and linear matrix inequalities (Gao *et al.* 2009). Moreover, the stochastic setting (Zhang & Tian 2009), the complex systems (Wang & Wang 2011), quantization (Carli *et al.* 2011) and asynchronous effect (Fang & Antsaklis 2008) are also interesting problems in the consensus study.

In this dissertation, for the consensus problem, the control laws will be designed to achieve consensus tracking in the last two chapters.

1.1.2.2 Formation control

Formation control objective is to control a group of agents such that desired formation shapes and cooperative tasks can be achieved. It has attracted much attention among the researchers community. The research in formation control has been motivated by various applications: a group of unmanned vehicles might be required to fly as a formation to provide covering surveillance of a region (see Fig. 1.12 (a)), a group of robots might be required to individually arrange themselves into a particular formation in order to avoid obstacles (see Fig. 1.12 (b)). In addition, formation control can be used in goal seeking, formation keeping, spacecraft docking, cooperative transportation, combat intelligence, reconnaissance and so on (Cao *et al.* 2013; Xue *et al.* 2010). The above applications verified that formation control are able to accomplish tasks more efficiently and more robustly. Due to broad applications and its advantages, formation control has been extensively studied by numerous researchers from various perspectives (Cao *et al.* 2013; Lin & Jia 2010; Xiao *et al.* 2009). In general, formation control can be divided into two types according to reference state: formation producing and formation tracking.

(a) (b)

Figure 1.12: Examples of formation (a) unmanned air vehicles for surveillance; (b) multiple robots for avoiding obstacles.

Formation producing: If multi-agents are not required to track a reference state, the formation problem is called formation producing.

Formation tracking: If multi-agent systems are required to track a reference state, the formation problem of multi-agent systems is called formation tracking.

For the formation producing, a simple example is also given in Fig. 1.13, where a desired formation shape is achieved without a reference state. For the formation tracking, Fig. 1.14 is shown to illustrate the tracking process.

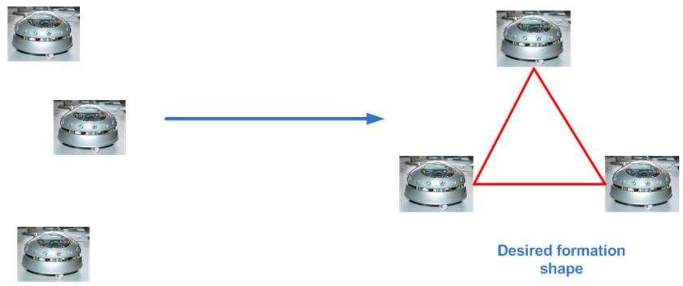


Figure 1.13: The problem of formation producing.

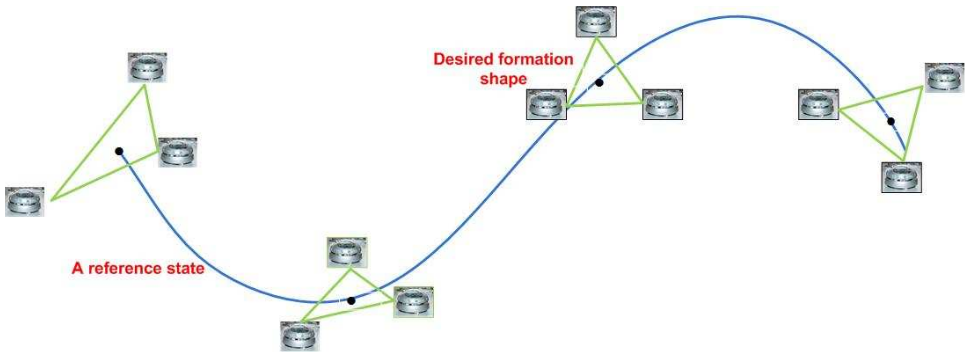


Figure 1.14: The problem of formation tracking.

As above stated, multi-agents are not required to track a reference state in the formation producing case. In recent years, the existing researches on formation producing aim at finishing formation behaviors by using some control laws. Up to now, there are many approaches were used for solving this problem, which mainly include **matrix theory**, **Lyapunov function method**, **graph rigidity** and **receding horizon method**.

The matrix theory approach (Lin *et al.* 2008; Ren 2008; Sepulchre *et al.* 2008) was used as in consensus problem. In which, some control laws for systems with single-integrator kinematics and double-integrator dynamics were investigated. In addition, some special nonlinear dynamics systems were also studied using the matrix theory. Matrix theory is a simple method for stability analysis of formation producing. However, it can't be applied in most nonlinear systems, therefore, Lyapunov approach was considered (Chen *et al.* 2014). In (Cucker & Dong 2010; Tanner *et al.* 2007), avoiding collision in flocking was studied, and the stabilization was mainly discussed. Dimarogonas & Kyriakopoulos 2007 considered an inverse agreement control law for multiple kinematic agents to force the team members to disperse in the workplace. The consensus producing with communication-delay and input delay was studied in (Meng *et al.* 2011) using Lyapunov method. Besides, motivated by the graph rigidity, formation producing was investigated to drive agents to the desired configuration by ensuring that a certain number of edge distances are identical to desired ones (Cao *et al.* 2011). Less information is required about edge distance using graph rigidity method compared with using other methods. At last, based on the optimization problem, receding horizon approach was used to solve formation producing by calculating some cost functions (Dunbar & Caveney 2012).

When multi-agents are required to track a reference state, the formation problem is called formation tracking. **The matrix approach** and **Lyapunov approach** were also used in formation tracking problem (Cao & Ren 2012; Do 2008; Fang & Antsaklis 2006; Lai *et al.* 2014; Wang *et al.* 2010a; Wen *et al.* 2012a), which will be introduced in detail in Chapter 4. In the matrix approach, formation tracking problem can be changed into a traditional stability problem by considering the error systems. However, to solve formation tracking of nonlinear multi-agent using Lyapunov approach, formation tracking is more difficult to be solved than formation producing, because the agents need to follow a reference state and maintain the desired formation geometric. Although formation producing is interesting in theory, it is more realistic to study formation tracking in the presence of a reference state.

In this dissertation, we will study formation producing with communication delay and absolute/relative damping in the first two parts of this dissertation. The reasons are given as follows. **Communication delay** is related to information transmission from one agent to another and affects the information state received from neighbors of each agent (Shen *et al.* 2012). Generally, the existence of communication delay is a source of instability and poor performance for a dynamic system. Therefore the analysis of communication delay is necessary. In addition, when agents work in complicated environments, there might exists fractional-order **absolute damping** or **relative damping**, which are aroused by absolute velocities or relative velocities between agents. Moreover, the fractional-order damping can improve the stability margin. Hence the control laws with communication delay and the damping is considered in the first two chapters of this dissertation. Their definitions will be given in Chapter 2.

For the formation tracking problem, we mentioned in the previous section that consensus problem is considered as part of formation control problem, which means that the latter results can be used in the consensus problem. But formation tracking demands both the tracking and desired formation keeping, therefore the results in consensus need to be extended to formation tracking problem. Hence, in the last two parts of this dissertation, based on the consensus results, we study formation tracking problem.

1.1.2.3 Task assignment

Task assignment refers to assigning task for a group of agents, which is also an important problem in distributed coordination, and this problem will be our future work. Its study mainly includes the following three directions: convergence control, scheduling and surveillance.

Convergence control is to allot the mobile sensors to maximizing the detection probability and minimizing the cost function. Actually, the convergence control can be viewed as an optimization problem (Choi & Horowitz 2010; Cortes 2010). Scheduling is to schedule a group of agent in a distributed manner. The study of this problem can be divided into two contents: sequence optimization and task allocation. The objective of the former is to optimize some metrics. For example, the total spending time is the metric in (Jin *et al.* 2006), where the optimal scheduling sequence was designed to estimate the metric. The objective of the latter is to assign tasks such that a group of agents can balance the total tasks (Reveliotis & Roszkowska 2011). Surveillance refers to monitoring a certain area using a group of agents more effectively than a single agent (Nigam & Kroo 2008; Pennisi *et al.* 2014). The potential applications of surveillance include board security, forest fire monitoring and oil spill patrolling.

1.1.2.4 Estimation

Estimation is also an interesting problem in distributed coordination, which will be studied in our future work. In the absence of global information, then, many estimation schemes are required. Generally, there are two steps to solve the distributed coordination (Charrow *et al.* 2013; Chen & Chen 2011): the first step is to design a local distributed estimator to estimate the global information asymptotically or in finite time. The second step is to design local controllers based on the local estimator to achieve the distributed coordination. For example, the estimation and control problem was studied with or without disturbance in (Lynch *et al.* 2008; Zhang & Leonard 2010). On the other hand, the estimation study is motivated to replace some expensive devices when agents work in some complex environment (Subbotin & Smith 2009; Yu 2010).

Among the above four directions in distributed coordination, this dissertation mainly consider two of them: consensus and formation control.

1.1.3 Fractional-Order Systems

The study of fractional-order systems has attracted an increasing interest since three hundred years (Miller & Ross 1993), In 1695, the prelude for studying fractional-order was starting from a letter, where Leibniz discussed the notion of fractional differentiation of non-integer order $\frac{1}{2}$. After that, Leibniz (Leibniz 1697) discussed the way for using fractional derivatives in finding the infinite product for $\frac{1}{2}\pi$. Then, for this problem, a lot of famous mathematicians participated in the discussion, For instance, Laplace considered the fractional derivatives by means of integrals in 1812. Fourier defined fractional operations using his function integral definition in 1822. Neils Henrik Abel used the fractional operators in finding the solutions of famous Tautochrone problem in 1823. Inspired by the above works, Liouville applied his definitions to the problems of potential theory and the definition led to wide discussion. To give a suitable definition for the study of fractional calculus, G. F. Bernhard, Grunwald and Caputo gave classic definitions respectively (Podlubny 1999).

However, the above investigation are mainly focus on the theory study, the real application of fractional calculus are difficult due to its unclear physical meanings and limited mathematical. Thanks to the development of mathematical theory and the computing technology in recent years, the fractional-order control systems have been widely investigated (N'Doye *et al.* 2013; NâĂŹDoye *et al.* 2013; Victor *et al.* 2015; Yousfi *et al.* 2013), and its applications have been also considered by researchers from different disciplines, such as fractances (Mehaute & Crepy 1983; Nakagawa & Sorimachi 1992), electrical circuits theory (Westerlund & Ekstam 1994), chaos theory (Bai & Yu 2010; Bai *et al.* 2012), physics (Ochoa-Tapia *et al.* 2007; Valdes-Parada *et al.* 2007), chemical mixing (Oldham & Spanier 1974), signal processing (Tseng 2007), mechatronics systems (Silva *et al.* 2004), biology (Cole 1933; Rihan 2013), engineering (Goodwine 2014; Vinagre *et al.* 2010) and so on. Specifically, the fractance (Mehaute & Crepy 1983), which has properties between resistance and capacitance, is an electrical circuit with fractional impedance. In addition, due to the memory property in fractional calculus, it is well used in capacitor theory (de Levie 1990; Westerlund & Ekstam 1994). In chaos theory, chaos occurs in integer systems of order 3 or more. With the introduction of fractional-order systems, some researchers study chaos in the system of total order less than 3 (Petráš & Vinagre 2002), such as the following Volta's system

$$\begin{cases} x_1^{0.98}(t) = x_1(t)19x_2(t)x_3(t)x_2(t), \\ x_1^{0.98}(t) = x_2(t)11x_1(t)x_1(t)x_3(t), \\ x_1^{0.98}(t) = 0.73x_3(t) + x_1(t)x_2(t) + 1, \end{cases} \tag{1.9}$$

and the synchronization of fractional-order chaotic system can show excellent application in security work. In physical, Méhauté 1990 verified that the current is proportional to the fractional derivative of voltage when fractal interface is put between a metal and an ionic medium. Podlubny 1999 verified that arbitrary order derivative and integrals are more suitable to describe properties of polymer materials. Especially, viscoelasticity is the property of material between purely elastic and pure fluid. For real materials, this property between stress and strain can be given by Hooke's law and Newton's law, but both have obvious disadvantages. So the fractional calculus (Meral *et al.* 2010; Müller *et al.* 2011) is introduced to describe this property: $\sigma(t) = ED_t^{\alpha}\varepsilon(t), 0 < \alpha < 1$. In addition, the fractional derivative can very well explain the anomalous diffuse phenomena in inhomogeneous media (El-Sayed 1996). In (Cole 1933), it proved that the membranes of cells in biological organism contain fractional electrical conductance. Moreover, fractional calculus was introduced in the engineering community to design a CRONE (Command Robust d'Ordre Non Entire) controller (Oustaloup 1995; Yousfi *et al.* 2014).

In engineering, the digital fractional-order controller was designed to control temperature (Petráš & Vinagre 2002). In addition, fractional-order hybrid control of robot manipulators were studied by Ferreira *et al.* 2008; Machado & Azenha 1998, where integer and fractional control laws were used, the results showed that the fractional-order control laws have superior performances. Besides, the fractional-order PID controller (Zhang *et al.* 2005) was used to control aerodynamic missile and good performance was got. It was shown that the changes of orders of fractional-order differentiation and integration can change frequency response curves of systems more subtly and flexibility (Zeng *et al.* 2002). Furthermore, the past behavior information can be kept inside the fractional differentiator (Huang *et al.* 2007; Zhang *et al.* 2005), which means that the history information of agents can affect their present and future states, this benefits for the control quality of systems. The accuracy and computational aspects of modeling a multi-robot system using fractional-order difference equation was investigated (Goodwine 2014). It was demonstrated that even for a relatively small system composed of simple elements with integer-order dynamics, the resulting relationship between the first and last generation of robots exhibited significant fractional-order effects. Moreover, it has been stated that the well-studied integer-order systems are just the special cases of the fractional-order ones (Liao *et al.* 2011; Tricaud & Chen 2010).

1.1.3.1 Fractional-order multi-agent systems

Note that most studies on distributed coordination of multi-agent systems are based on integer-order dynamics. However, the fact that many phenomena in nature can't be explained in integer-order dynamics, while some more well reflections to system properties can be given by fractional-order systems. Fractional-order calculus concerns arbitrary order of differentiation and integration, which can be viewed as an extension of integer calculus.

With the development of fractional calculus as above statement, the distributed coordination of fractional-order multi-agent systems has also attracted great interest in recent years. Its applications include the synchronized motion of agents in complex environments such as macromolecule fluids and porous media (Powell 1970; Sabatier *et al.* 2007), specifically, flocking movement and food searching by means of the individual secretions and microbial, submarine underwater robots in the bottom of the sea with a large number of microorganisms and viscous substances, unmanned aerial vehicles operating in an environment where the influence of particles in air can't be ignored (e.g., high-speed flight in dust storm, rain, or snow), and ground vehicles moving on top of carpet, sand, muddy road, or grass (Cao & Ren 2010a; Oldham & Spanier 1974).

The consensus producing of fractional-order systems was proposed for the first time by Cao *et al.* 2010, where the convergence speed of consensus for fractional-order systems and that for integer-order systems are compared. Lately, consensus of fractional-order multi-agent systems with time delay was investigated. The input delay was studied by Liu *et al.* 2012, where a tight upper bound of the input delay that could be tolerated in fractional-order multi-agent systems was given. Consensus with communication delay over directed topologies (Liu *et al.* 2012; Shen & Cao 2011; Shen *et al.* 2012; Yang *et al.* 2014) was considered and the bound on the communication delay was obtained exactly according to the Nyquist stability theorem. In addition, the consensus problem for a class of fractional-order uncertain multi-agent systems is studied by Li 2012; Li *et al.* 2014, where an observer-type consensus protocol and the robust stabilizing controllers were proposed to achieve consensus. Besides this sliding mode control method (Ferrara *et al.* 2007; Vignoni *et al.* 2012; Zhao *et al.* 2012) were used to solve consensus producing and consensus tracking of fractional-order multi-agent. Especially, the accuracy and computational aspects of modeling a multi-robot system using fractional-order difference equation was investigated (Goodwine 2014). It was demonstrated that even for a relatively small system comprised of simple elements with integer-order dynamics, the resulting relationship between the first and last generation of robots exhibited significant fractional-order effects.

Based on the advantages of fractional-order systems and its investigations on fractional-order multi-agent systems, we focus on the study of distributed coordination of fractional-order multi-agent systems in this dissertation.

1.2 Contributions and Outline of Dissertation

This dissertation presents distributed coordination of fractional-order multi-agents systems under fixed directed communication graph, the consensus problem and the formation control problem are investigated for distributed coordination. The main contributions are summarized as follows.

Chapter 2: Chapter 2 investigates the formation producing of fractional-order multi-agent systems with absolute damping and communication-delay. The contribution of this chapter is threefold. Firstly, the fractional-order multi-agent systems and the control algorithm are given, and according to the vector conversion, the nonlinear systems are changed into linear systems. Then, using the matrix theory, graph theory and the frequency domain analysis, the results are given in the following theorem, it is shown that the formation control will be achieved if the following conditions are guaranteed: $\alpha \in (0, 2]$, the weighted communication topology has a directed spanning tree and all the roots of characteristic equation have negative real parts or 0. Finally, the simulation results are respectively provided to validate the validity of our theoretical analysis. Comparing with existing works listed in the literature, this chapter has the following advantages: Firstly, in contrast to most papers which study the distributed multi-agent coordination systems with linear dynamics, in this chapter the nonlinear multi-agent system with fractional-order absolute damping is considered. Secondly, it is well known that time delay is very important in the practical applications, however, there are few relative results on formation control of fractional-order multi-agent systems with time delay. Hence, the formation producing of fractional-order multi-agent systems with communication delay is considered in this chapter. Finally, different from existing results on the stability analysis of equilibrium points using Lyapunov method, in this chapter, the frequency-domain analysis method is used to consider the stability analysis of equilibrium points. In fact, for the fractional-order dynamical systems, since there are substantial differences between fractional-order differential systems and integer-order differential ones, it is very difficult and inconvenient to construct Lyapunov functions.

Chapter 3: In Chapter 2, the formation producing with absolute damping is discussed. The classic integer multi-agent means that all agents achieve formation asymptotically with zero final velocities. However, in some scenarios, it might be desirable that all agents achieve formation and move as a group, instead of rendezvous at a stationary point. In this case, only relative measurements (position or vehicles) are needed, it is more difficult to finish formation producing with relative damping. Based on above idea, when agents work in a complex environment, we aim to propose a control law with relative damping for formation producing of fractional-order multi-agent systems. The contributions of this chapter are given as follows: Firstly, a distributed formation control law with communication delay is given under directed interaction graph. Secondly, stability conditions for formation producing of fractional-order multi-agent systems with relative damping and communication delay are given using the frequency-domain analysis method. Finally, to illustrate the validity of the obtained results, several simulations are presented based on predictor-corrector method. Comparing with existing works in the literature, this chapter has the same advantages as the results in chapter 2. Meanwhile, different from the above chapter, agents can converge to stationary point, in this chapter, agents can move as a group in the presence of communication delay.

Chapter 4: Note that chapter 2 and chapter 3 study formation producing without a reference state, the final target value to be reached is an inherent point or trajectory. However, it is desirable that the states of all agents can asymptotically track a reference state, representing the state of common interest for all other agents, which is required in many practical applications, examples include formation flying, body guard, and coordinated tracking applications. Therefore, this chapter mainly investigates consensus tracking. Firstly, a common control law is proposed, and a theorem is given to verify the validity of the control law when a communication graph includes a directed spanning tree. Secondly, a control law based on error predictor is proposed, and its validity is also verified according to a theorem when a communication graph has a directed spanning tree. The convergence speed of fractional-order multi-agent systems based on the above control laws is then compared. It is verified that the convergence speed is faster using the control law based on error predictor than using the common one. Thirdly, the control law based on error predictor is extended to solve the formation tracking problem. Finally, several simulations are presented to verify the validity of the obtained results. Comparing with existing papers results, this chapter has the following differences. Firstly, in contrast to the studies without a reference state, we considered the consensus of multi-agent systems with a reference state. Secondly, the consensus of multi-agent systems is studied based on fractional-order systems instead of integer order ones. Two effective control laws are given. Finally, the convergence speed is compared based on the proposed two control laws.

Chapter 5: In chapter 4, we study the consensus tracking of fractional-order multi-agent systems. Note that all agents have access to the reference state, but in practice, the reference state for the whole team might only be available to a single or a portion of agents. Therefore, this chapter continues considering the consensus with a reference state, where only a portion of agents have access to the reference state. Firstly, a consensus control law is given to solve the consensus problem of fractional-order multi-agent systems with a constant reference state. However, it is shown that the consensus control law cannot guarantee consensus with a time-varying reference state. Then, a general control law and a particular one for consensus with a time-varying reference state of fractional-order multi-agent systems are proposed. It is shown that if the directed communication graph has a directed spanning tree, all agents can track the time-varying reference state using the proposed control laws. Next, the above control laws are extended to solve the formation tracking problem. Finally, several simulations are presented respectively to verify the validity of the obtained results. Comparing with existing works, the results of this chapter have the following differences. Firstly, the consensus with a reference state in these works is based on integer-order multi-agent systems, while this chapter considers the consensus with a reference state and formation tracking based on fractional-order multi-agent systems. Secondly, in existing papers, it is required that the reference state is available to all agents, in this chapter, only a portion of the agents in the group can receive the information of time-varying reference state directly.

Conclusion and Perspectives: In this chapter, the results are summarized and several possible directions for our future research are identified.

Publications:

[1] ***Jing Bai***, Guoguang Wen, Ahmed Rahmani and Yongguang Yu, *Distributed formation control of fractional-order multi-agent systems with absolute damping and communication delay*, International Journal of Systems Science, doi:10.1080/00207721.2014.998411, 2014.

[2] ***Jing Bai***, Guoguang Wen, Ahmed Rahmani and Yongguang Yu, *Consensus with a reference state for fractional-order multi-agent systems*, International Journal of Systems Science, doi:10.1080/00207721.2015.1056273, 2015.

[3] ***Jing Bai***, Guoguang Wen, Ahmed Rahmani and Yongguang Yu, *Formation Tracking of Fractional-Order Multi-Agent Systems Based on Error Predictor*, The 27th Chinese Control and Decision Conference, 23. May. 2015.

[4] ***Jing Bai***, Guoguang Wen, Ahmed Rahmani and Yongguang Yu, *Distributed formation control of fractional-order multi-agent systems with relative damping and communication delay* , **Submitted** to International Journal of Control, Automation and Systems, February. 2015.

[5] ***Jing Bai***, Guoguang Wen, Ahmed Rahmani and Yongguang Yu, *Consensus with a reference state for fractional-order multi-agent systems based on error predictor*, **Submitted** to IET Control Theory & Applications, March. 2015.

Chapter 2

Formation Producing of Fractional-Order Multi-agent Systems with Absolute Damping and Communication Delay

Contents

2.1 Introduction

From the definition in Chapter 1, formation producing means controlling a group of agents such that desired formation shapes in the absence of a reference state. So far, numerous results have been shown on formation producing (Cao *et al.* 2013; Fischer *et al.* 1995; Liu *et al.* 2007). Note that most of the cited literatures on formation control of multi-agent systems are based on integer-order dynamics. However, many phenomena in nature can't be explained using integer-order dynamics, while better description of the system properties can be given using fractional-order systems. Hence, we will study formation producing of fractional-order multi-agent systems.

It has shown in chapter 1 that consensus producing can be viewed as a part of formation one, up to now, there are many studies on the consensus producing problem. Consensus producing of fractional-order systems was proposed for the first time by Cao *et al.* 2010, where the convergence speed of consensus between fractional-order systems and for integer-order ones is compared. A tight upper bound on the input delay that can be tolerated in the fractional-order multi-agent systems was obtained (Liu *et al.* 2012). The consensus producing for a class of fractional-order uncertain multi-agent systems is studied (Li 2012; Li *et al.* 2014), where an observer-type consensus protocol was proposed (Li 2012), and the robust stabilizing controllers were derived by using linear matrix inequality approach and matrix's singular value decomposition. Besides this sliding mode control method (Ferrara *et al.* 2007; Vignoni *et al.* 2012) was used to solve consensus producing of fractional-order multi-agent systems. However, there are few results on formation producing of fractional-order multi-agent systems (Cao & Ren 2010a).

In many physical systems, when the information exchange between agents is required, the time delay is ubiquitous due to several reasons (Cao *et al.* 2013): (1) limited communication speed when information transmission exists; (2) extra time required by the sensors to get the measurement information; (3) computation time required for generating the control inputs; (4) execution time required for the inputs to being acted. Up to now there are many results (Darouach 2006; Ezzine *et al.* 2013; Lu *et al.* 2012; Wang *et al.* 2012; Wang & Wu 2012) on integer-order multi-agent systems with time delay, but few results (Liu *et al.* 2012; Yang *et al.* 2014) on fractional-order multi-agent systems with time delay. In reality, researchers discuss communication delay and input delay. Communication delay is related to information transmission from one agent to another and affects the information state received from neighbors of each agent (Shen *et al.* 2012). Generally, the existence of communication delay is a source of instability and poor performance for a dynamic system. Therefore the analysis of communication delay is necessary.

It is well known that the control effect is better using the nonlinear control laws, but all the above results are about the linear control laws of fractional-order multi-agent systems, hence the nonlinear control laws with communication delay is studied in our work. Using nonlinear control laws with absolute/relative damping, the formation control of fractional-order multi-agent systems was studied in (Cao & Ren 2010a), and sufficient conditions on the network topology were given to ensure the formation control. In applications, there might exist fractional-order absolute damping when agents work in complicated environments, and the fractional-order damping can improve the stability margin. Therefore, the absolute damping is also considered in our work.

Comparing with existing works in the literatures, this chapter has the following advantages: Firstly, in contrast to most papers (Cao *et al.* 2010; Zhao *et al.* 2013) which study the distributed multi-agent coordination systems with linear dynamics, in this chapter the nonlinear multi-agent system with fractional-order absolute damping is considered. Secondly, while it is well known that time delay is very important in practical applications, there are few related papers (Liu *et al.* 2012; Shen & Cao 2011; Yang *et al.* 2014) on formation control of fractional-order multi-agent systems with time delay. Hence, the formation producing of fractional-order multi-agent systems with communication delay is considered in this chapter. Finally, different from existing results (Peng *et al.* 2013a; Wen *et al.* 2012b) on the stability analysis of equilibrium points using Lyapunov method, in this chapter, the frequency-domain analysis method is used to consider the stability analysis of equilibrium points. In fact, for the fractional-order dynamical systems, since there are substantial differences between fractional-order differential systems and integer-order ones, it is very difficult and inconvenient to construct Lyapunov functions. In this chapter, the formation producing of fractional-order multi-agent systems with absolute damping and communication delay is considered (Bai *et al.* 2015). This chapter is organized as follows: Firstly, a distributed formation control law with communication delay is given under directed interaction topology. Secondly, sufficient conditions on the stability for the fractional-order multi-agent systems with absolute damping and communication delay are given using the frequency-domain analysis method to ensure achieving the formation producing. Finally, several simulations are presented based on the numerical method of predictor-corrector to illustrate the effectiveness of the obtained results.

2.2 Preliminaries

Before formulating our problem, we introduce the concepts of fractional derivative, communication delay and the absolute damping.

Fractional derivative has two mostly used definitions: Riemanne-Liouville and Caputo definition. The former is mainly used for the theory study in pure mathematics. The latter can provide clear physical interpretation for the initial conditions, hence, this definition is commonly used in many real applications. We will use the Caputo fractional derivative defined as follows (Podlubny 1999)

$$ {}_a^C D_t^{\alpha} x(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{x^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, \tag{2.1} $$

where α is an arbitrary positive real number, m is the first integer which is not less than α, i.e., $m-1 < \alpha \leq m$, functions $x(t)$ has m continuous derivatives for $t \geq 0$. ${}_a^C D_t^{\alpha}$ denotes the Caputo derivative with an order α, and $\Gamma(\cdot)$ is the Gamma function given by $\Gamma(p) = \int_0^{+\infty} t^{p-1} e^{-t} dt$ which has the following property

$$ \Gamma(p+1) = x\Gamma(p), \tag{2.2} $$

with p being an arbitrary real number.

Assumption 2.1 *For the fractional derivative problem, we just study the domain* $(0, 1]$ *for its order, because any order can be changed into considering the domain* $(0, 1]$ *(Podlubny 1999).*

The Laplace transform of Caputo fractional derivative is

$$ L\{{}_a^C D_t^{\alpha} x(t)\} = s^{\alpha} X(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1} x^{(k)}(0), \tag{2.3} $$

where $X(s) = L\{x(t)\} = \int_0^{+\infty} e^{-st} x(t) dt$. Hence, the Laplace transform of Caputo fractional derivative can be written as follow

$$L\{{}_a^C D_t^\alpha x(t)\} = \begin{cases} s^\alpha X(s) - s^{\alpha-1}x(0), & \alpha \in (0,1] \\ s^\alpha X(s) - s^{\alpha-1}x(0) - s^{\alpha-2}\dot{x}(0), & \alpha \in (1,2] \end{cases} \tag{2.4}$$

where $x(0^-) = x_{\epsilon\to 0^-}(\epsilon)$ and $\dot{x}(0^-) = \dot{x}_{\epsilon\to 0^-}(\epsilon)$.

In this thesis, in order to simulate the fractional-order multi-agent systems, the numerical method of predictor-corrector (Bhalekar 2013; Bhalekar & Daftardar-gejji 2011; Diethelm 1997; Diethelm *et al.* 2002) is used. The detailed content of this method (Bhalekar 2013) is given in Appendix A.

Communication delay describes the time of transmitting information from origin to destination. Specifically, if it takes time $t_{i,j}$ for agent i to receive information from agent j. For example, a single integrator based on consensus control law with communication delay is described as follows

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{i,j}(x_j(t-\tau_{i,j}) - x_i(t)), i, j \in N, i \neq j \tag{2.5}$$

where $x_i(t) \in R$ represents the state of agent i, $a_{i,j}$ is the $(i,j)th$ entry of the adjacency matrix A. $N = (1, 2, \cdots, n)$ denotes the set of the indexes of agents. An interpretation of (2.5) is that agent i receives information from agent j and uses data $x_j(t-\tau_{i,j})$ instead of $x_j(t)$ due to the communication delay.

Communication delay $\tau_{i,j}$ might be constant or time-varying, for the case of time varying, the problem is more complex and not considered in this thesis, it will be our future work. We only consider constant communication delay.

Absolute damping velocity control law is proposed for second order multi-agent systems with communication delay taking the form of

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t-\tau_{i,j}) - x_i(t)] - c \cdot \dot{x}_i(t), i, j \in N, i \neq j \tag{2.6}$$

where c is positive constant representing the control gain, $c \cdot \dot{x}_i(t)$ is the absolute damping item. In (Qiao & Sipahi 2012), the authors verified that stability region of communication delay grows with increasing the absolute damping, which means that absolute damping can increases the communication delay margin. Due to this advantage, the control law with absolute damping is used in fractional-order multi-agent systems with communication delay, which is given as the following form

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t-\tau_{i,j}) - x_i(t)] - c \cdot x_i^{(\alpha/2)}(t), i, j \in N, i \neq j \tag{2.7}$$

where $x_i(t), x_j(t) \in R$ and $u_i(t) \in R$ represent the state and control input of agent i. $N = (1, 2, \cdots, n)$ denotes the set of the indexes of agents, $i, j \in N$ $x_i^{(\alpha/2)}(t)$ is the $\alpha/2$th Caputo derivative of $x_i(t)$. $a_{i,j}$ is the $(i,j)th$ entry of the adjacency matrix A, $\tau_{i,j}$ represents the communication delay from agent j to agent i.

2.3 Problem Description

To study the formation producing of fractional-order multi-agent systems with communication delay and absolute damping, in this section, we introduce the fractional-order multi-agent systems and the problem objective. For simplicity, we give the following notation to describe the fractional derivative.

Notation 2.2 *Since only the Caputo fractional derivative is used in this thesis, a simple notation $x^{(\alpha)}(t)$ is used to denote ${}_a^C D_t^{\alpha} x(t)$.*

The fractional-order systems for n agents can be given as

$$x_i^{(\alpha)}(t) = u_i(t), i \in N, \tag{2.8}$$

where $x_i(t) \in R$ and $u_i(t) \in R$ represent the state and control input of agent i. $N = (1, 2, \cdots, n)$ denotes the set of the indexes of agents and $x_i^{(\alpha)}(t)$ is the αth Caputo derivative of $x_i(t)$. For the order α, we have the following assumption.

Assumption 2.3 *Assume $\alpha \in (0, 2]$ in this chapter based on Assumption 2.1, then its order can be changed into the domain $(0, 1]$ when we use the frequency-domain analysis method.*

In reality, the possible tasks could range from exploration of unknown environments where the desired state deviation (e.g. distance) among agents could potentially reduce the exploration time, and could avoid collision among agents, such as multi-robots, multi-vehicles and so on. Hence, we use the term formation control to refer to the behavior that a group of multi-agent systems reaches the desired state deviation via a local interaction. The objective of formation producing is given as follows.

Definition 2.4 *The formation producing of multi-agent systems is to design a control input $u_i(t)$, such that the states of all the agents $x_i(t)$ satisfy the following equation*

$$\lim_{t\to+\infty}(x_i(t) - x_j(t)) = \delta_{ij}, i, j \in N, i \neq j \tag{2.9}$$

where $\delta_{ij} = \delta_i - \delta_j$ denotes the desired state deviation between the agent i and the agent j.

2.4 Distributed Control Law with Communication Delay and Absolute Damping

Based on the problem description, a control law with communication and absolute damping is designed to solve the formation producing, at the same time, sufficient conditions are given to guarantee the effectiveness of the control law.

Consider the following control law with absolute damping and communication delay as

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t-\tau_{i,j}) - x_i(t) + \delta_{ij}] - c \cdot x_i^{(\alpha/2)}(t), i, j \in N, i \neq j \tag{2.10}$$

where $a_{i,j}$ is the $(i,j)th$ entry of the adjacency matrix A, $\tau_{i,j}$ represents the communication delay from agent j to agent i, $c \in R^+$, c is a positive constant representing the control gain, and $x_i^{(\alpha/2)}(t)$ represents the absolute damping.

Substituting control law (2.10) into Eq. (2.8), the system can be written as

$$\tilde{x}_i^{(\alpha)}(t) = \sum_{j=1}^{n} a_{i,j}[\tilde{x}_j(t-\tau_{i,j}) - \tilde{x}_i(t)] - c \cdot \tilde{x}_i^{(\alpha/2)}(t), i, j \in N, i \neq j \tag{2.11}$$

where $\tilde{x}_i(t) = x_i(t) - \delta_i$, $\tilde{x}_j(t-\tau_{i,j}) = x_j(t-\tau_{i,j}) - \delta_j$.

Let $y_1(t) = \tilde{x}_1(t), \cdots, y_n(t) = \tilde{x}_n(t)$, $y_{n+1}(t) = \tilde{x}_1^{(\alpha/2)}(t), \cdots, y_{2n(t)} = \tilde{x}_n^{(\alpha/2)}(t)$, $\beta = \alpha/2 \in (0, 1]$.

According to the above hypothesis, the multi-agent systems (2.11) with n agents can be expressed as follows

$$\begin{cases} y_1^{(\beta)}(t) = y_{n+1}(t), \\ \vdots \\ y_n^{(\beta)}(t) = y_{2n}(t), \\ y_{n+1}^{(\beta)}(t) = \sum_{j=1}^{n} a_{1,j}(y_j(t-\tau_{1,j}) - y_1(t)) - c \cdot y_{n+1}(t), \\ y_{n+2}^{(\beta)}(t) = \sum_{j=1}^{n} a_{2,j}(y_j(t-\tau_{2,j}) - y_2(t)) - c \cdot y_{n+2}(t), \\ \vdots \\ y_{2n}^{(\beta)}(t) = \sum_{j=1}^{n} a_{n,j}(y_n(t-\tau_{n,j}) - y_n(t)) - c \cdot y_{2n}(t). \end{cases} \tag{2.12}$$

In this chapter, the formation producing problem for multi-agent systems (2.11) becomes the stability problem for linear fractional-order system with communication delay, that is to say, if the solution of systems (2.12) is stable as $t \to \infty$, then the formation producing with communication delay can be achieved.

Theorem 2.5 *Suppose directed communication graph G has a directed spanning tree, and $\alpha = 2\beta \in (0, 2]$, then formation producing of the systems (2.8) can be asymptotically achieved using the control law (2.10) if the following condition is satisfied:*
all the roots of the characteristic equation $det(\Delta(s)) = 0$ have negative real parts or equate to zero, where $\Delta(s)$ is a characteristic matrix as follows

$$\Delta(s) = \tag{2.13}$$
$$\left(\begin{array}{c|ccc|c|ccc} s^\beta & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & s^\beta & \cdots & 0 & 0 & -1 & \cdots & 0 \\ \vdots & & & & \vdots & & & \\ 0 & 0 & \cdots & s^\beta & 0 & 0 & \cdots & -1 \\ \sum_{j=1}^{n} a_{1,j} & -a_{1,2}e^{-s\tau_{1,2}} & \cdots & -a_{1,n}e^{-s\tau_{1,n}} & s^\beta + c & 0 & \cdots & 0 \\ -a_{2,1}e^{-s\tau_{2,1}} & \sum_{j=1}^{n} a_{2,j} & \cdots & -a_{2,n}e^{-s\tau_{2,n}} & 0 & s^\beta + c & \cdots & 0 \\ \vdots & & & & \vdots & & & \\ -a_{n,1}e^{-s\tau_{n,1}} & -a_{n,2}e^{-s\tau_{n,2}} & \cdots & \sum_{j=1}^{n} a_{n,j} & 0 & 0 & \cdots & s^\beta + c \end{array}\right).$$

Proof. First, any equilibrium y^* of Eq. (2.12) is given in Ref. (Xu & Li 2013) as

$$Fy^* = \left[\begin{array}{cc} 0_{n\times n} & I_n \\ -L & -cI_n \end{array} \right] y^* = 0, \tag{2.14}$$

where L is the Laplace matrix of the communication control. Which means that y^* is a right eigenvector of F associated with the zero eigenvalue. According to matrix theory, the matrices F and $\bar{F}$ has same eigenvalues, where $\bar{F}$ is given as Eq. 2.15

$$\bar{F} = \left[\begin{array}{cc} -(1/c)L & I_n \\ 0 & -cI_n \end{array} \right]. \tag{2.15}$$

Obviously, the eigenvalue zero of matrix $\bar{F}$ is simple, hence the eigenvalue zero of matrix F is also simple. If the eigenspace associated with zero eigenvalue is one-dimensional, according to Lemma 1.6, there exists $b \in R$ such that $y^* = [b, \cdots, b, 0, \cdots, 0]$.

The linear fractional-order equation (2.12) has a non-zero equilibrium when $b \neq 0$, and this equilibrium can be moved to the origin by the translation transform $\tilde{y}_i(t) = y_i(t) - b, i \in N$, and $\tilde{y}_i(t) = y_i(t), i \notin N$. Then, Eq. (2.12) can be written as the following form

$$\begin{cases} \tilde{y}_1^{(\beta)}(t) = \tilde{y}_{n+1}(t), \\ \vdots \\ \tilde{y}_n^{(\beta)}(t) = \tilde{y}_{2n}(t), \\ \tilde{y}_{n+1}^{(\beta)}(t) = \sum_{j=1}^{n} a_{1,j}(\tilde{y}_j(t-\tau_{1,j}) - \tilde{y}_1(t)) - c \cdot \tilde{y}_{n+1}(t), \\ \tilde{y}_{n+2}^{(\beta)}(t) = \sum_{j=1}^{n} a_{2,j}(\tilde{y}_j(t-\tau_{2,j}) - \tilde{y}_2(t)) - c \cdot \tilde{y}_{n+2}(t), \\ \vdots \\ \tilde{y}_{2n}^{(\beta)}(t) = \sum_{j=1}^{n} a_{n,j}(\tilde{y}_j(t-\tau_{n,j}) - \tilde{y}_n(t)) - c \cdot \tilde{y}_{2n}(t). \end{cases} \tag{2.16}$$

Next, the stability of the zero solution of Eq. (2.16) is to be discussed in the frequency domain. Taking Laplace transform on both sides of Eq. (2.16) gives

$$\begin{cases} s^\beta \tilde{Y}_1(s) - s^{\beta-1}\tilde{y}_1(0) & = \tilde{Y}_{n+1}(s), \\ \vdots & \\ s^\beta \tilde{Y}_n(s) - s^{\beta-1}\tilde{y}_n(0) & = \tilde{Y}_{2n}(s), \\ s^\beta \tilde{Y}_{n+1}(s) - s^{\beta-1}\tilde{y}_{n+1}(0) & = \sum_{j=1}^n a_{1,j}(e^{-s\tau_{1,j}}\tilde{Y}_j(s) - \tilde{Y}_1(s)) - c\cdot \tilde{Y}_{n+1}(s), \\ s^\beta \tilde{Y}_{n+2}(s) - s^{\beta-1}\tilde{y}_{n+2}(0) & = \sum_{j=1}^n a_{2,j}(e^{-s\tau_{2,j}}\tilde{Y}_j(s) - \tilde{Y}_2(s)) - c\cdot \tilde{Y}_{n+2}(s), \\ \vdots & \\ s^\beta \tilde{Y}_{2n}(s) - s^{\beta-1}\tilde{y}_{2n}(0) & = \sum_{j=1}^n a_{n,j}(e^{-s\tau_{n,j}}\tilde{Y}_j(s) - \tilde{Y}_n(s)) - c\cdot \tilde{Y}_{2n}(s), \end{cases} \tag{2.17}$$

which can be rewritten in the following compact matrix form

$$(s^\beta I + \tilde{F}(s))\tilde{Y}(s) = s^{\beta-1}\tilde{y}(0), \tag{2.18}$$

where

$$\tilde{F}(s) = \tag{2.19}$$
$$\left(\begin{array}{c|c|ccc|c} 0 & 0 & 0 & \cdots & 0 & -1 \cdots 0 \\ & \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \cdots -1 \\ \sum_{j=1}^n a_{1,j} & -a_{1,2}e^{-s\tau_{1,2}} & -a_{1,3}e^{-s\tau_{1,3}} & \cdots & -a_{1,n}e^{-s\tau_{1,n}} & c \cdots 0 \\ -a_{2,1}e^{-s\tau_{2,1}} & \sum_{j=1}^n a_{2,j} & -a_{2,3}e^{-s\tau_{2,3}} & \cdots & -a_{2,n}e^{-s\tau_{2,n}} & 0 \cdots 0 \\ & \vdots & & & & \vdots \\ -a_{n,1}e^{-s\tau_{n,1}} & -a_{n,2}e^{-s\tau_{n,2}} & -a_{n,3}e^{-s\tau_{n,3}} & \cdots & \sum_{j=1}^n a_{n,j} & 0 \cdots c \end{array}\right)$$

$\tilde{Y}_i(s)$ is the Laplace transform of $\tilde{y}_i(t)$ with $\tilde{Y}_i(s) = L(\tilde{y}_i(t))$, $\tilde{y}_i(0)$ being the initial values of $\tilde{y}_i(t), i = 1, 2, \ldots, 2n$. $\tilde{Y}(s) = (\tilde{Y}_1(s), \cdots, \tilde{Y}_{2n}(s))^T$, $\tilde{y}(0) = (\tilde{y}_1(0), \cdots, \tilde{y}_{2n}(0))^T$.

Define the characteristic matrix of Eq. (2.17) as $\Delta(s) = s^\beta I + \tilde{F}(s)$, which is given in Theorem 2.5. It then follows that the corresponding characteristic equation can be written as follows

$$det(s^\beta I + \tilde{F}(s))) = 0. \tag{2.20}$$

Multiplying $s^{1-\beta}$ on both sides of Eq. (2.18), we have

$$(s^{1-\beta}(s^{\beta}I + \tilde{F}(s)))\tilde{Y}(s) = (sI + s^{1-\beta}\tilde{F}(s))\tilde{Y}(s) = \tilde{y}(0). \tag{2.21}$$

According to the matrix theory, it is easy to verify that $det(sI+s^{1-\beta}\tilde{F}(s)) = 0$ has the same non-zero solutions with $det(s^{\beta}I+\tilde{F}(s))) = 0$. Next, we only consider the solutions of $det(sI + s^{1-\beta}\tilde{F}(s)) = 0$.

Obviously, $s = 0$ is a solution of the characteristic equation $det(sI+s^{1-\beta}\tilde{F}(s)) = 0$. Therefore, if all the roots of the characteristic equation are on the left-half plane or $s = 0$, then the zero solution of Eq. (2.16) is asymptotically stable (Shen & Cao 2011), i.e.,

$$\lim_{t\to+\infty} \tilde{y}_i(t) = 0, i = 1, 2, \cdots, 2n. \tag{2.22}$$

Due to $\lim_{t\to+\infty} \tilde{y}_i(t) = \lim_{t\to+\infty} y_i(t) - b_i = 0$ and $\lim_{t\to+\infty} y_i = \lim_{t\to+\infty} \tilde{x}_i(t) = \lim_{t\to+\infty} x_i(t) - \delta_i, i \in N$, then the following results can be obtained

$$\lim_{t\to+\infty} y_i(t) = b_i + \lim_{t\to+\infty} \tilde{y}_i(t), i \in N, \tag{2.23}$$

and

$$\lim_{t\to+\infty} x_i(t) = \lim_{t\to+\infty} y_i(t) + \delta_i, i \in N. \tag{2.24}$$

Also due to $b_i = b_j = b, i, j \in N$, then

$$\lim_{t\to+\infty} (x_i(t) - x_j(t)) = b_i - b_j + \delta_i - \delta_j = \delta_{ij}. \tag{2.25}$$

Hence, the definition of the formation producing is satisfied, the proof of this theorem is completed. ■

Remark 2.6 *If the conditions of Theorem 2.5 are satisfied, the zero solution of Eq. (2.16) is asymptotically stable, that is to say, the formation producing can be achieved using the control law (2.10).*

Remark 2.7 *Through above discussions, Theorem 2.5 gives conditions to judge whether the formation producing with communication delay can be achieved using the control law (2.10), when β and c are given, and the communication delay* *is limited, if all the roots of* $det(\Delta(s))$ *are negative or equate to zero, then the control law is effective for Eq. (2.8). But the method is not practical when a large number of agents are considered.*

Remark 2.8 *When* $\tau_{i,j} = \tau = 0$*, the existing formation producing of fractional-order systems with absolute damping studied in (Cao & Ren 2010a) can be viewed as a special case of this chapter. When* $c = 0$ *and* $\tau_{i,j} = \tau = 0$*, the results in (Cao et al. 2010) can also be viewed as a special case.*

Remark 2.9 $\alpha = 2i, i \in N$*, can be chosen, which means that integer-order systems are the special cases of fractional-order ones.*

Remark 2.10 *When* $\delta_i = 0$*, the formation producing problem turns into the consensus producing problem.*

2.5 Simulations

In this section, several simulation results are presented to illustrate the effectiveness of the control law proposed in this chapter.

For simplicity, we first consider a group of two agents with communication delay, and their communication graphs that have spanning trees are shown in Fig. 2.1, an arrow from j to i denotes that agent i can receive information from agent j. According to the Theorem 2.5, we can calculate the characteristic equation of the multi-agent system with two agents as follows

$$det\left(\Delta(s)\right) = det\begin{pmatrix} s^{\beta} & 0 & -1 & 0 \\ 0 & s^{\beta} & 0 & -1 \\ 1 & -e^{-s\tau} & s^{\beta}+1 & 0 \\ 0 & 0 & 0 & s^{\beta}+1 \end{pmatrix} = 0, \tag{2.26}$$

which can be rewritten as the following form

$$s(s^{3\beta} + 2s^{2\beta} + 2s^{\beta} + 1) = 0 \tag{2.27}$$

Note that all the roots of the characteristic Eq. (2.27) are not relative with communication delay. Then we can obtain that all roots of characteristic equation (2.27) have negative real parts -0.174 or equation to zero, that satisfies the conditions in the theorem. Choose $\alpha = 2\beta = 1.8$. The initial states of the two agents are chosen as $x_1(0) = 3, x_2(0) = 2, x_1^{(\alpha/2)}(0) = 2, x_2^{(\alpha/2)}(0) = 2$. Here, for simplicity, we choose $\delta_{ij} = \delta_i - \delta_j = 0$, $c = 1$. Fig. 2.2 shows the state responses of the two agents with time delay $\tau = 0.1$, and Fig. 2.3 shows the state responses of the two agents with time delay $\tau = 1$. From above state responses, we can see that formation producing has been achieved, and the stability are not relative with communication delay. Figs. 2.4 and 2.5 show that the formation producing can also be achieved when $\alpha = 2\beta = 2$. The initial states of the two agents are given as $x_1(0) = -3, x_2(0) = 3, x_1^{(\alpha/2)}(0) = 5, x_2^{(\alpha/2)}(0) = -1$. All roots of characteristic equation (2.27) have negative real parts -1 and -0.5, which satisfy the conditions of the theorem. This simulation verifies that formation producing of integer-order systems are the special cases of the formation producing of fractional-order ones.

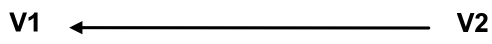


Figure 2.1: Directed communication graph for two agents.

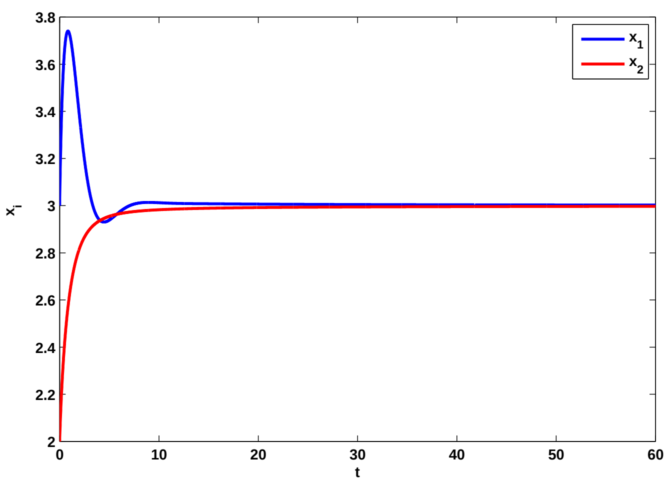


Figure 2.2: State responses of the two agents with communication delay $\alpha = 2\beta = 1.8$, $\tau = 0.1$.

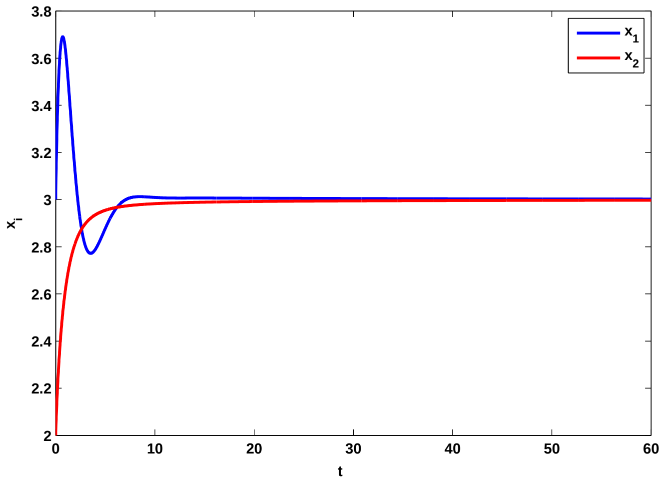


Figure 2.3: State responses of the two agents with communication delay $\alpha = 2\beta = 1.8$, $\tau = 1$.

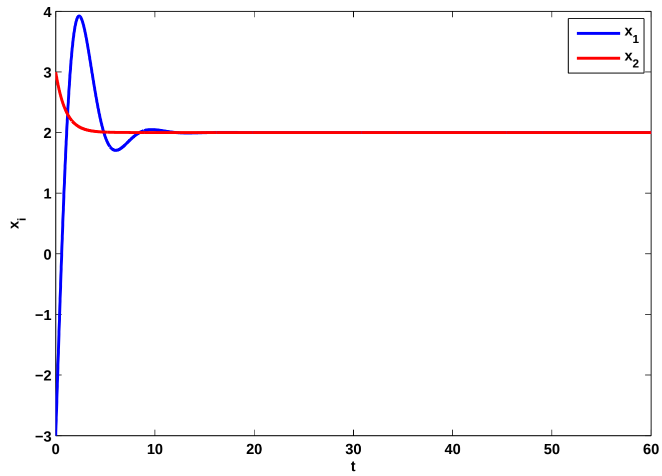


Figure 2.4: State responses of the two agents with communication delay $\alpha = 2\beta = 2$, $\tau = 0.1$.

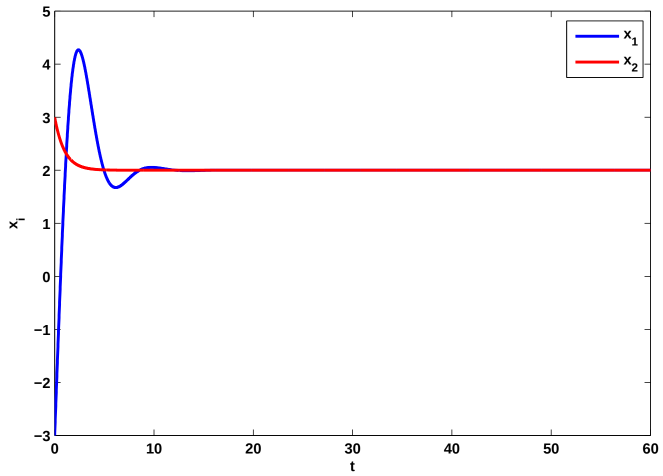


Figure 2.5: State responses of the two agents with communication delay $\alpha = 2\beta = 2$, $\tau = 1$.

In the following, let us consider a group of three agents with communication delay in two dimensional space, and their communication graph has spanning trees, which is shown in Fig. 2.6. Choose $c = 1$, $\beta = 0.9$, $\tau = 0.1$, and $\delta_i = (0, 0)^T$, the initial states of the three agents in the two dimensional space are chosen as $(x_1(0), y_1(0))^T = (-2, 3)^T$, $(x_2(0), y_2(0))^T = (2, -4)^T$, $(x_3(0), y_3(0))^T = (5, -2)^T$, $(x_1^{(\alpha/2)}(0), y_1^{(\alpha/2)}(0))^T = (-1, 0)^T$, $(x_2^{(\alpha/2)}(0), y_2^{(\alpha/2)}(0))^T = (3, 1)^T$, $(x_3^{(\alpha/2)}(0), y_3^{(\alpha/2)}(0))^T = (5, 2)^T$. According to the Theorem 2.5, the characteristic equation of the multi-agent system with three agents can be calculated as follows

$$det\Delta(s) = \tag{2.28}$$
$$det\begin{pmatrix} s^\beta & 0 & 0 & -1 & 0 & 0 \\ 0 & s^\beta & 0 & 0 & -1 & 0 \\ 0 & 0 & s^\beta & 0 & 0 & -1 \\ 1 & -e^{-s\tau} & 0 & s^\beta + 1 & 0 & 0 \\ 0 & 1 & -e^{-s\tau} & 0 & s^\beta + 1 & 0 \\ -e^{-s\tau} & 0 & 1 & 0 & 0 & s^\beta + 1 \end{pmatrix} = 0,$$

which can be rewritten as the following form

$$s^{6\beta} + 3s^{5\beta} + 6s^{4\beta} + 7s^{3\beta} + 6s^{2\beta} + 3s^\beta + 1 - e^{-3s\tau} = 0 \tag{2.29}$$

For solving the nonlinear equation (2.29), a numerical method is used in Ref.

Powell (1970). Let

$$\begin{cases} L_1 = s^{6\beta} + 3s^{5\beta} + 6s^{4\beta} + 7s^{3\beta} + 6s^{2\beta} + 3s^{\beta} + 1, \\ L_2 = e^{-3s\tau}. \end{cases} \tag{2.30}$$

From Fig. 2.7, all roots s are located in the interval $(-10, 70)$, and we search the solutions in $[-15, 100]$. Fig. 2.8 shows the distribution for the real parts of s in Eq. (2.29). We can see that all the real parts of the solutions are not more than 0, which means that the real parts of s are negative or equate to zero and satisfies the condition in the theorem. Fig. 2.9 shows the x-state responses of the three agents under the control law (2.10), and Fig. 2.10 shows the y-state responses of three agents under the control law (2.10). Fig. 2.11 shows position trajectories of the three agents. It can be noted from Figs. 2.9-2.11 that the consensus producing is achieved. When $\delta_1 = (0, 0)^T$, $\delta_2 = (0.5, 0.5)^T$, $\delta_3 = (-0.5, 0.5)^T$, from Fig. 2.13, the formation producing can be achieved and the desired formation geometric as Fig. 2.12 is achieved.

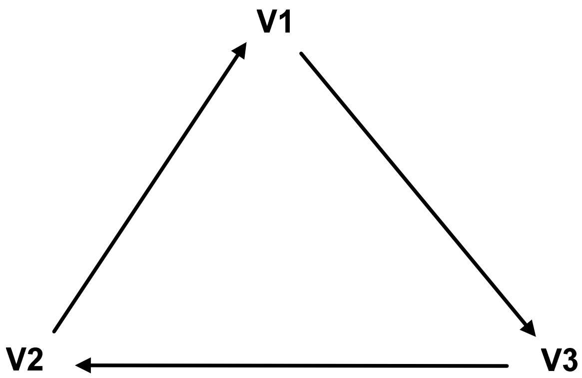


Figure 2.6: Directed communication graph for three agents.

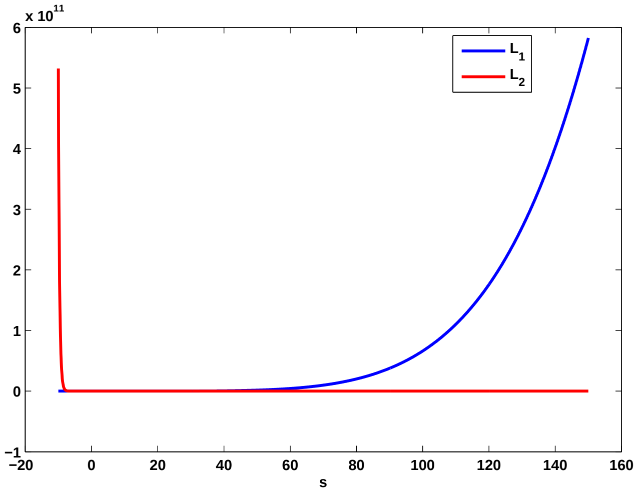


Figure 2.7: Curve diagram of L_1 and L_2 based on Eq. (2.29) for the case of three agents.

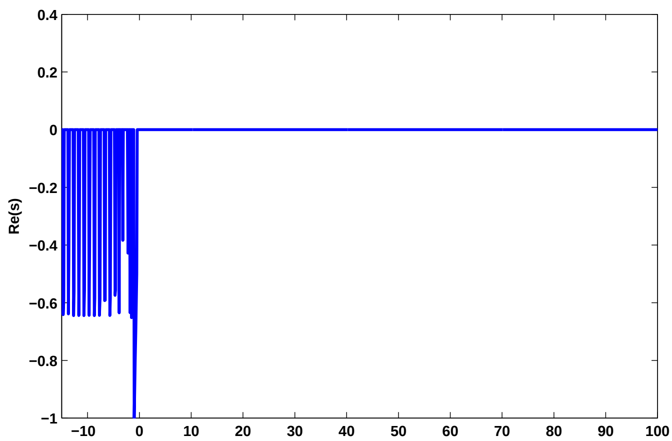


Figure 2.8: The distribution for the real parts of the roots of Eq. (2.29).

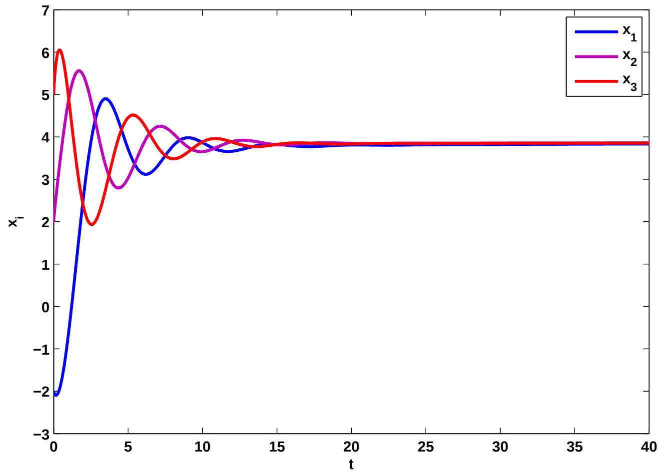


Figure 2.9: X-state responses of the three agents under control law (2.10).

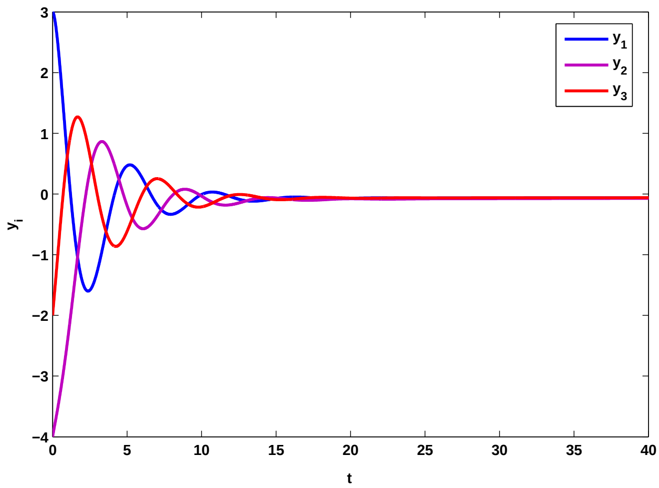


Figure 2.10: Y-state responses of the three agents under control law (2.10).

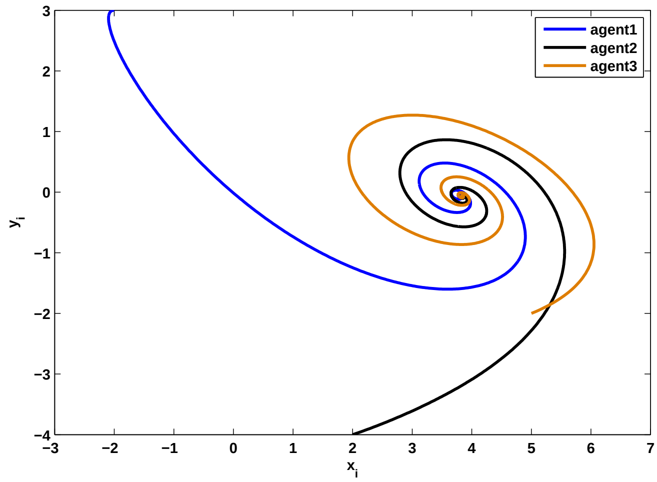


Figure 2.11: Position trajectories of the three agents.

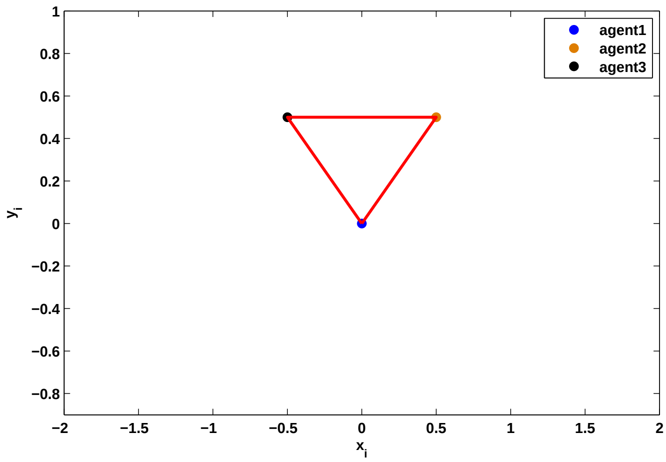


Figure 2.12: The desired formation geometric form for three agents.

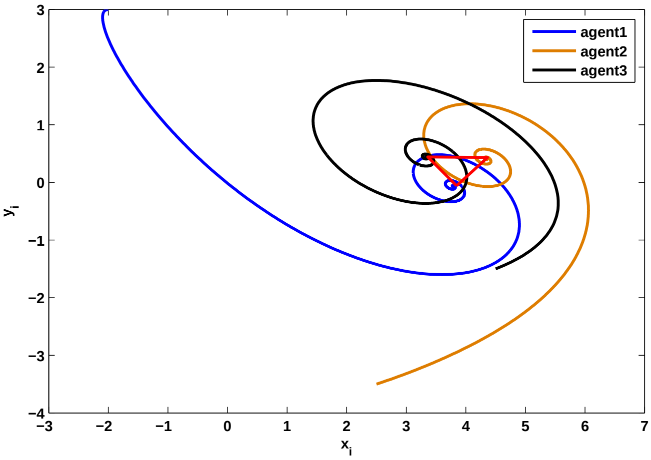


Figure 2.13: Position formation trajectories of the three agents.

Next, let us consider a group of six agents indexed by 1, 2, ..., 6, respectively. The directed communication graph G that has a directed spanning tree is shown in Fig. 2.14. According to Theorem 2.5, the characteristic equation of the multi-agent system with six agents can be calculated as follows

$$
\begin{aligned}
& det\Delta(s) = \\
& det \begin{pmatrix}
s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & -e^{-s\tau} & 0 & 0 & s^\beta+1 & 0 & 0 & 0 & 0 & 0 \\
-e^{-s\tau} & 1 & 0 & 0 & 0 & 0 & 0 & s^\beta+1 & 0 & 0 & 0 & 0 \\
0 & -e^{-s\tau} & 1 & 0 & 0 & 0 & 0 & 0 & s^\beta+1 & 0 & 0 & 0 \\
0 & 0 & -e^{-s\tau} & 1 & 0 & 0 & 0 & 0 & 0 & s^\beta+1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -e^{-s\tau} & 0 & 0 & 0 & 0 & s^\beta+1 & 0 \\
-e^{-s\tau} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & s^\beta+1
\end{pmatrix} \\
& = 0.
\end{aligned} \tag{2.31}
$$

For solving the nonlinear equation, the same numerical method Powell (1970) is used. From Fig. 2.15, all roots of s are located in the interval $(-80, 80)$, and it is enough to search the solutions of Eq 2.31 in $(80, 80)$. Fig. 2.16 shows the distribution for the real parts of s, we can see that all the real parts of the solutions are not more than 0 when the error is in the accepted range, which means that the real parts of s are negative or equate to zero, the conditions in the theorem are satisfied. The initial states of the six agents in two dimensional space are chosen as $(x_1(0), y_1(0))^T = (3.5, 3)^T$, $(x_2(0), y_2(0))^T = (2, -4)^T$,$(x_3(0), y_3(0))^T = (2, -2)^T$, $(x_4(0), y_4(0))^T = (6, 0)^T$, $(x_5(0), y_5(0))^T = (-1, 0)^T$,$(x_6(0), y_6(0))^T = (0.5, 1)^T$, $(x_1^{(\alpha/2)}(0), y_1^{(\alpha/2)}(0))^T = (0, 0)^T$, $(x_2^{(\alpha/2)}(0), y_2^{(\alpha/2)}(0))^T = (0, -1)^T$, $(x_3^{(\alpha/2)}(0), y_3^{(\alpha/2)}(0))^T = (0, 2)^T$, $(x_4^{(\alpha/2)}(0), y_4^{(\alpha/2)}(0))^T = (0, -0.5)^T$, $(x_5^{(\alpha/2)}(0), y_5^{(\alpha/2)}(0))^T = (1, 0)^T$, $(x_6^{(\alpha/2)}(0), y_6^{(\alpha/2)}(0))^T = (-1, 1)^T$, and $c = 1$, $\beta = 0.85$, $\tau = 0.1$, $\delta_i = (0, 0)^T$. Fig. 2.17 shows the x-state responses of six agents under control law (2.10), and Fig. 2.18 shows the y-state responses of six agents. Fig. 2.19 shows the position trajectories of the six agents. It can be noted from Figs. 2.17- 2.19 that the consensus producing is achieved. When $\delta_1 = (-0.5, \frac{\sqrt{3}}{2})^T$, $\delta_2 = (-1, 0)^T$, $\delta_3 = (-0.5, -\frac{\sqrt{3}}{2})^T$, $\delta_4 = (0.5, -\frac{\sqrt{3}}{2})^T$, $\delta_5 = (1, 0)^T$, $\delta_6 = (0.5, \frac{\sqrt{3}}{2})^T$ are chosen, from Fig. 2.21, the formation producing is achieved, and the desired formation geometric form as Fig. 2.20 is also attained.

Above simulations verify that consensus producing can be viewed as a part of formation producing and the above theories on formation producing are effective.

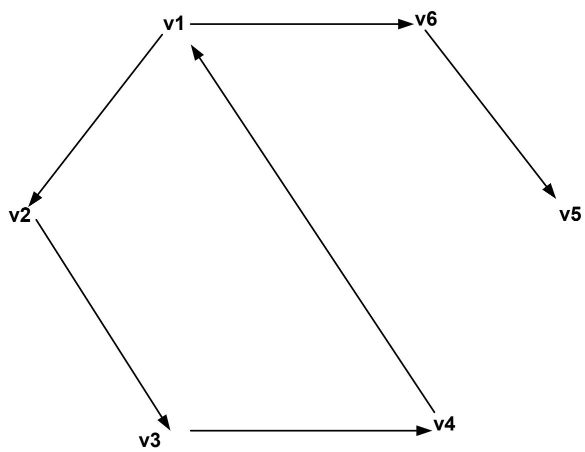


Figure 2.14: Directed communication graph for six agents.

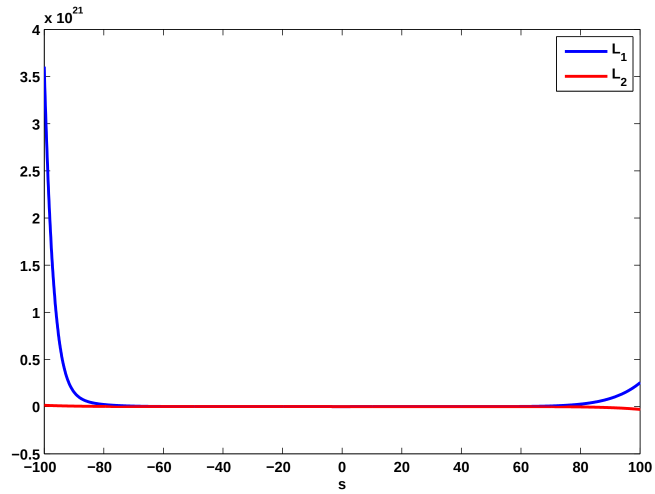


Figure 2.15: Curve diagram of L_1 and L_2 based on Eq. (2.31) for the case of six agents.

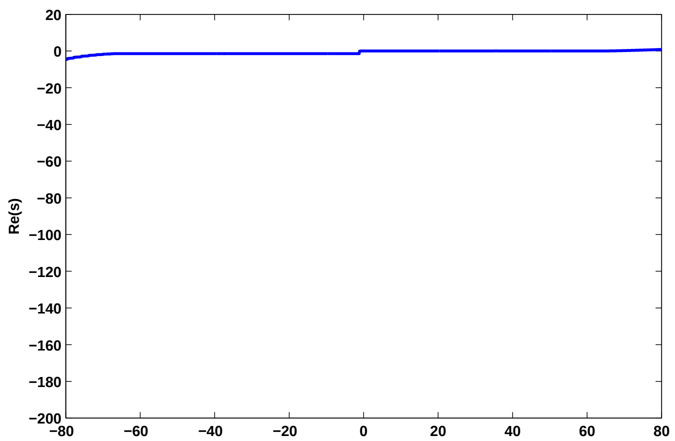


Figure 2.16: The distribution for the real parts of the roots of Eq. (2.31).

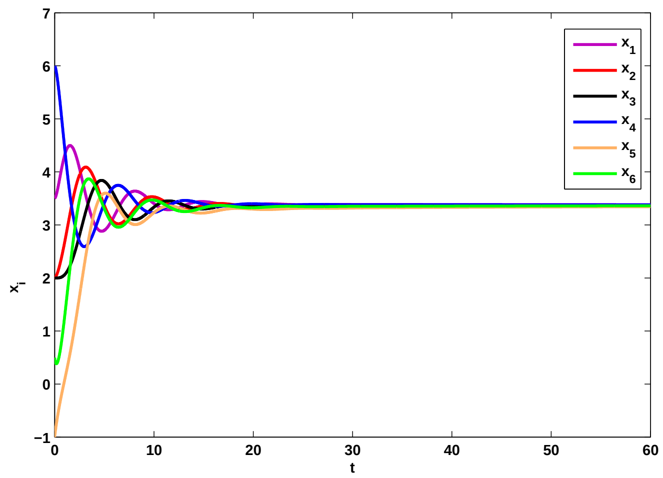


Figure 2.17: X-state responses of the six agents under control law (2.10).

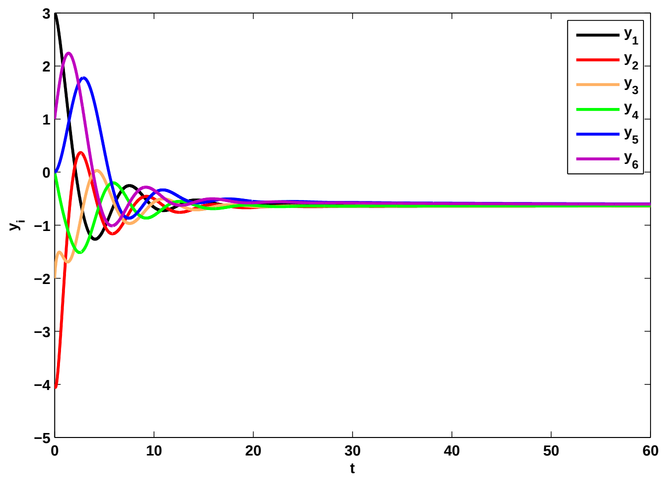


Figure 2.18: Y-state responses of the six agents under control law (2.10).

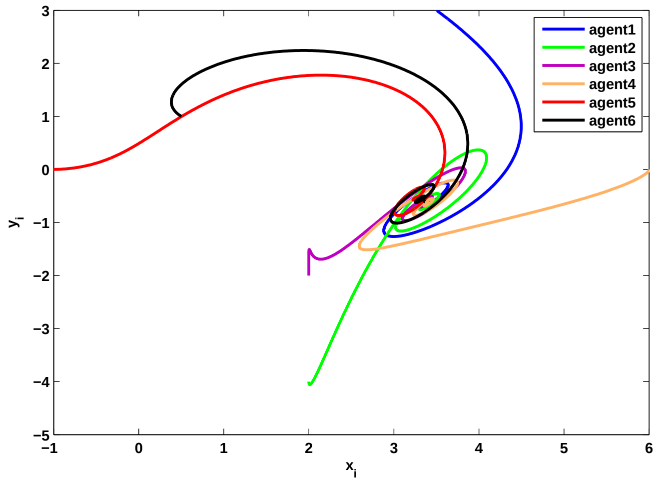


Figure 2.19: Position trajectories of the six agents.

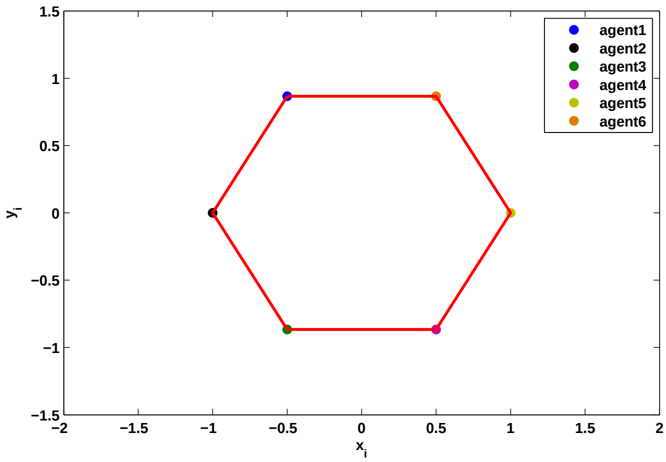


Figure 2.20: The desired formation geometric form for six agents.

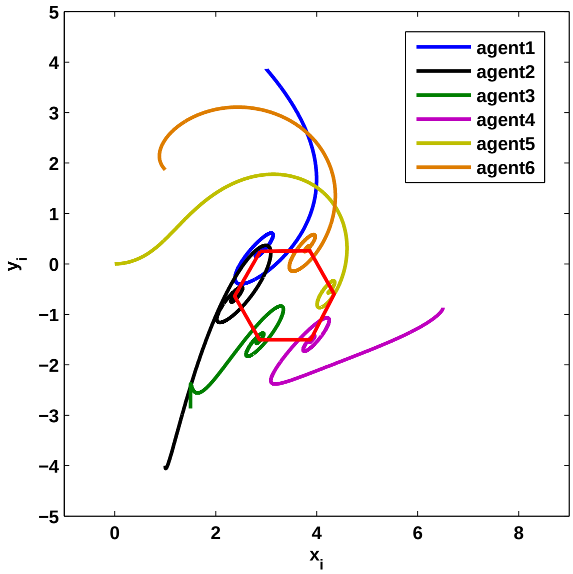


Figure 2.21: Position formation trajectories of the six agents.

At last, in the case of 6 agents, we choose a smaller $\beta = \alpha/2 = 0.5$, and keep other parameters. We use the obtained results and above method, From Fig. 2.22, all roots of s are located in the interval $(-15, 15)$, Fig. 2.23 shows the distribution for the real parts of s, we can see that all the real parts of the solutions are not more than 0 when the error is in the accepted range, which means that the real parts of s are negative or equate to zero, the conditions in the theorem are satisfied. From Fig. 2.24, the formation producing is achieved, but we find that the convergence speed is slower than the above case, this problem will be our future work.

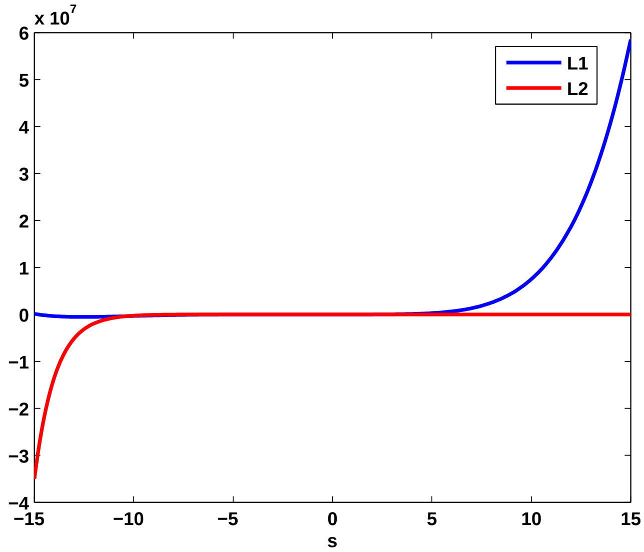


Figure 2.22: Curve diagram in the case of $\beta = \alpha/2 = 0.5$.

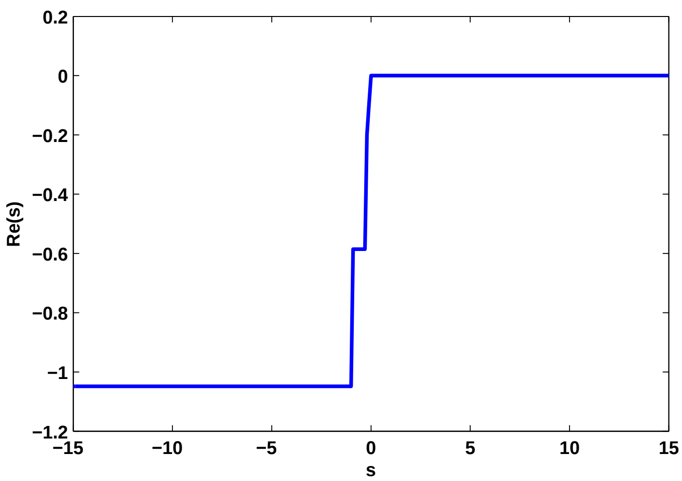


Figure 2.23: The distribution for the real parts of the roots.

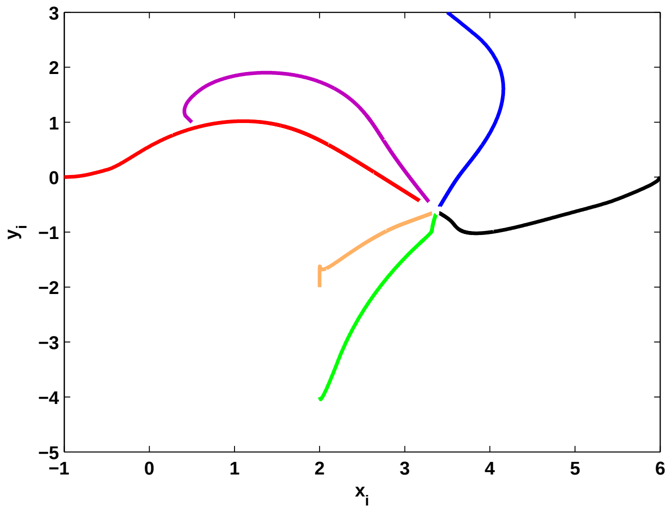


Figure 2.24: Consensus trajectories of the six agents.

2.6 Conclusion

In this chapter we have studied formation producing of fractional-order multi-agent systems with absolute damping and communication delay. Firstly, fractional-order multi-agent systems and a control law have been given. Then, using the matrix theory, graph theory and the frequency domain analysis, the conditions of formation producing have been shown in the theorem. Finally, the simulation of the directed communication graph with two agents has been given to verify that the integer-order systems are the special cases of fractional-order systems. Furthermore, the achieving of the formation producing of three agents and six agents have been provided to validate our theoretical analysis. From the simulations, the method to judge the formation producing is complex when a large number of agents are considered, hence, more simple methods to judge the problem will be our future work.

Chapter 3

Formation Producing of Fractional-Order Multi-Agent Systems with Relative Damping and Communication Delay

Contents

3.1 Introduction

In Chapter 2, formation producing with absolute damping was discussed. In the classic integer multi-agent, the formation producing in Chapter 2 means that all agents achieve formation asymptotically with zero final velocities. However, in some scenarios, it might be desirable that all agents achieve formation and move as a group, instead of rendezvous at a stationary point. In this case, only relative measurements (position or velocity) are needed (Mei *et al.* 2014). Pursuing this idea, when agents are more suitable to be described by the fractional-order systems in an environment, we propose a control law with relative damping for formation producing of fractional-order multi-agent systems.

Comparing with existing works in the literatures, this chapter has the following advantages: Firstly, different from the result in Chapter 2, agents converge to stationary points, agents can move as a group in the presence of communication delay in this chapter. Secondly, in contrast to most papers (Cao *et al.* 2010; Zhao *et al.* 2013) which study the distributed multi-agent coordination systems with linear dynamics, the nonlinear multi-agent systems are proposed. Finally, communication delay is included in our study, while there is little work published on this (Liu *et al.* 2012; Shen & Cao 2011; Yang *et al.* 2014). This chapter is organized as follows: Firstly, a distributed formation control law with communication delay is given under directed communication graph. Secondly, stability conditions for formation producing of fractional-order multi-agent systems with relative damping and communication delay are established using the frequency-domain analysis method. Finally, to illustrate the effectiveness of the obtained results, several simulations are presented based on predictor-corrector method.

3.2 Preliminaries

Before formulating our problem, we introduce the relative damping, and the concepts of fractional derivative and communication delay can be found in chapter 2.

Relative damping velocity control law is proposed for second-order multi-agent systems taking the form as

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}\{[x_j(t-\tau_{i,j}) - x_i(t)] + c \cdot [\dot{x}_j(t) - \dot{x}_i(t)]\}, i, j \in N, i \neq j \qquad (3.1)$$

where $\tau_{i,j}$ represents the communication delay from agent j to agent i. c is positive constant representing the control gain, $[\dot{x}_j(t) - \dot{x}_i(t)]$ is the relative damping item. In (Qiao & Sipahi 2012), it verified that the stable region of communication delay enlarges with increasing c. Due to above advantage, the control law with relative damping is used in fractional-order multi-agent systems with communication delay, which is given as the following form

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}\{[x_j(t-\tau_{i,j}) - x_i(t)] + c \cdot [x_j^{(\alpha/2)}(t) - x_i^{(\alpha/2)}(t)]\}, i, j \in N, i \neq j \tag{3.2}$$

where $x_i(t) \in R$ and $u_i(t) \in R$ represent the state and control input of agent i, respectively. $N = (1, 2, \cdots, n)$ denotes the set of the indexes of agents, $x_i^{(\alpha)}(t)/2$ is the $\alpha/2$th Caputo derivative of $x_i(t)$. $a_{i,j}$ is the $(i, j)th$ entry of the adjacency matrix A, $\tau_{i,j}$ represents the communication delay from agent j to agent i.

3.3 Problem Description

In this section, we introduce the fractional-order multi-agent systems and the problem objective. $\alpha \in (0, 2]$ is also assumed throughout this chapter, and a simple notation $x^{(\alpha)}(t)$ is used to denote the Caputo fractional derivative mentioned in Chapter 1.

The fractional-order systems for n agents can be given as

$$x_i^{(\alpha)}(t) = u_i(t), \alpha \in (0, 2], i \in N \tag{3.3}$$

where $x_i(t) \in R$ and $u_i(t) \in R$ represent the state and control input of agent i, respectively. $N = (1, 2, \cdots, n)$ denotes the set of the indexes of agents.

The definition of formation producing was given in Definition 2.4

$$\lim_{t\to+\infty}(x_i(t) - x_j(t)) = \delta_{ij},\ \ i, j \in N, i \neq j \tag{3.4}$$

where $\delta_{ij} = \delta_i - \delta_j$ denotes the desired state deviation between the agent i and the agent j.

3.4 Distributed Control Law with Communication Delay and Relative Damping

Consider the following control law with relative damping and communication delay as

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}\{[x_j(t-\tau_{i,j}) - x_i(t) + \delta_{ij}] + c \cdot [x_j^{(\alpha/2)}(t-\tau_{i,j}) - x_i^{(\alpha/2)}(t)]\}, i \neq j \tag{3.5}$$

where $i, j \in N$, $a_{i,j}$ is the $(i,j)th$ entry of the adjacency matrix A, $\tau_{i,j}$ represents the communication delay from agent j to agent i, c is a positive constant representing the control gain, and $(x_j^{(\alpha/2)}(t-\tau_{i,j}) - x_i^{(\alpha/2)}(t))$ represents the relative damping.

Substituting control law (3.5) into Eq. (3.3), the system can be written as

$$\tilde{x}_i^{(\alpha)}(t) = \sum_{j=1}^{n} a_{i,j}\{[\tilde{x}_j(t-\tau_{i,j}) - \tilde{x}_i(t)] + c \cdot [\tilde{x}_j^{(\alpha/2)}(t-\tau_{i,j}) - \tilde{x}_i^{(\alpha/2)}(t)]\}, \tag{3.6}$$

where $i, j \in N$, $i \neq j$, $\tilde{x}_i(t) = x_i(t) - \delta_i$, $\tilde{x}_j(t-\tau_{i,j}) = x_j(t-\tau_{i,j}) - \delta_j$, c and $a_{i,j}$ have the same definitions as the above classic integer systems.

Let $y_1(t) = \tilde{x}_1(t), \cdots, y_n(t) = \tilde{x}_n(t)$, $y_{n+1}(t) = \tilde{x}_1^{(\alpha/2)}(t), \cdots, y_{2n(t)} = \tilde{x}_n^{(\alpha/2)}(t)$, $\beta = \alpha/2 \in (0,1]$, $Y(0) = \tilde{X}(0)$ and $Y^{(\beta)}(0) = \tilde{X}^{(\alpha/2)}(0)$.

According to the above hypothesis, the multi-agent systems (3.6) with n agents can be expressed as follows

$$\begin{cases} y_1^{(\beta)}(t) = y_{n+1}(t), \\ \vdots \\ y_n^{(\beta)}(t) = y_{2n}(t), \\ y_{n+1}^{(\beta)}(t) = \sum_{j=1}^{n} a_{1,j}\{(y_j(t-\tau_{1,j}) - y_1(t)) + c \cdot (y_{n+j}(t-\tau_{1,j}) - y_{n+1}(t))\}, \\ y_{n+2}^{(\beta)}(t) = \sum_{j=1}^{n} a_{2,j}\{(y_j(t-\tau_{2,j}) - y_2(t)) + c \cdot (y_{n+j}(t-\tau_{2,j}) - y_{n+2}(t))\}, \\ \vdots \\ y_{2n}^{(\beta)}(t) = \sum_{j=1}^{n} a_{n,j}\{(y_n(t-\tau_{n,j}) - y_n(t)) + c \cdot (y_{n+j}(t-\tau_{n,j}) - y_{2n}(t))\}. \end{cases} \tag{3.7}$$

In this chapter, the formation producing problem for multi-agent systems (3.6) changes into the stability problem of linear fractional-order system with communication delay, that is to say, if the solution of systems (3.7) is stable as $t \to \infty$, then the formation producing with communication delay can be achieved.

Theorem 3.1 *Suppose that directed communication graph G has a directed spanning tree, and $\alpha = 2\beta \in (0, 2]$, then the formation producing of the systems (3.3) can be asymptotically achieved using the control law (3.5) if the following condition is satisfied:*

all the roots of the characteristic equation det$(\Delta(s)) = 0$ have negative real parts or $s = 0$, where $\Delta(s)$ is a characteristic matrix as follows

$$\Delta(s) = \tag{3.8}$$
$$\left(\begin{array}{c|ccc|ccc}
s^\beta & 0 & \cdots & 0 & -1 & \cdots & 0 \\
0 & s^\beta & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & & & & \vdots & & \\
0 & 0 & \cdots & s^\beta & 0 & \cdots & -1 \\
\sum_{j=1}^n a_{1,j} & -a_{1,2}e^{-s\tau_{1,2}} & \cdots & -a_{1,n}e^{-s\tau_{1,n}} & c \cdot \sum_{j=1}^n a_{1,j} + s^\beta & \cdots & -c \cdot a_{1,n}e^{-s\tau_{1,n}} \\
-a_{2,1}e^{-s\tau_{2,1}} & \sum_{j=1}^n a_{2,j} & \cdots & -a_{2,n}e^{-s\tau_{2,n}} & -c \cdot a_{2,1}e^{-s\tau_{2,1}} & \cdots & -c \cdot a_{2,n}e^{-s\tau_{2,n}} \\
\vdots & & & & \vdots & & \\
-a_{n,1}e^{-s\tau_{n,1}} & -a_{n,2}e^{-s\tau_{n,2}} & \cdots & \sum_{j=1}^n a_{n,j} & -c \cdot a_{n,1}e^{-s\tau_{n,1}} & \cdots & c \cdot \sum_{j=1}^n a_{n,j} + s^\beta
\end{array}\right).$$

Proof. First, any equilibrium y^* of Eq. (3.7) satisfies the equation (Bhalekar 2013)

$$Fy^* = \begin{bmatrix} 0_{n\times n} & I_n \\ -L & -c \cdot L \end{bmatrix} y^* = 0, \tag{3.9}$$

where L is the Laplace matrix of the communication control. According to the results in (Cao & Ren 2010a), $y^* = [b, \cdots, b, 0, \cdots, 0]$.

The linear fractional-order equations (3.7) has a non-zero equilibrium when $b \neq 0$, and this equilibrium can be moved to the origin by a translation $\tilde{y}_i(t) = y_i(t) - b, i \in N$, and $\tilde{y}_i(t) = y_i(t), i \notin N$. Then Eq. (3.7) can be written as the following form

$$\begin{cases} \tilde{y}_1^{(\beta)}(t) = \tilde{y}_{n+1}(t), \\ \vdots \\ \tilde{y}_n^{(\beta)}(t) = \tilde{y}_{2n}(t), \\ \tilde{y}_{n+1}^{(\beta)}(t) = \sum_{j=1}^{n} a_{1,j}\{(\tilde{y}_j(t-\tau_{1,j}) - \tilde{y}_1(t)) + c \cdot (\tilde{y}_{n+j}(t-\tau_{1,j}) - \tilde{y}_{n+1}(t))\}, \\ \tilde{y}_{n+2}^{(\beta)}(t) = \sum_{j=1}^{n} a_{2,j}\{(\tilde{y}_j(t-\tau_{2,j}) - \tilde{y}_2(t)) + c \cdot (\tilde{y}_{n+j}(t-\tau_{2,j}) - \tilde{y}_{n+2}(t)\}, \\ \vdots \\ \tilde{y}_{2n}^{(\beta)}(t) = \sum_{j=1}^{n} a_{n,j}\{(\tilde{y}_j(t-\tau_{n,j}) - \tilde{y}_n(t)) + c \cdot (\tilde{y}_j(t-\tau_{n,j}) - \tilde{y}_{2n}(t))\}. \end{cases} \tag{3.10}$$

The stability of the zero solution of Eq. (3.10) is to be discussed in the frequency domain. Taking Laplace transform on both sides of Eq. (3.10) gives

$$\begin{cases} s^\beta \tilde{Y}_1(s) - s^{\beta-1}\tilde{y}_1(0) & = \tilde{Y}_{n+1}(s), \\ \vdots & \\ s^\beta \tilde{Y}_n(s) - s^{\beta-1}\tilde{y}_n(0) & = \tilde{Y}_{2n}(s), \\ s^\beta \tilde{Y}_{n+1}(s) - s^{\beta-1}\tilde{y}_{n+1}(0) & = \sum_{j=1}^{n} a_{1,j}\{(e^{-s\tau_{1,j}}\tilde{Y}_j(s) - \tilde{Y}_1(s)) + c \cdot (e^{-s\tau_{1,j}}\tilde{Y}_{n+j}(s) \\ & -\tilde{Y}_{n+1}(s))\}, \\ s^\beta \tilde{Y}_{n+2}(s) - s^{\beta-1}\tilde{y}_{n+2}(0) & = \sum_{j=1}^{n} a_{2,j}\{(e^{-s\tau_{2,j}}\tilde{Y}_j(s) - \tilde{Y}_2(s)) + c \cdot (e^{-s\tau_{2,j}}\tilde{Y}_{n+j}(s) \\ & -\tilde{Y}_{n+2}(s))\}, \\ \vdots & \\ s^\beta \tilde{Y}_{2n}(s) - s^{\beta-1}\tilde{y}_{2n}(0) & = \sum_{j=1}^{n} a_{n,j}\{(e^{-s\tau_{n,j}}\tilde{Y}_j(s) - \tilde{Y}_n(s)) + c \cdot (e^{-s\tau_{n,j}}\tilde{Y}_{n+j}(s) \\ & -\tilde{Y}_{2n}(s))\}, \end{cases} \tag{3.11}$$

which can be rewritten in the following compact matrix form

$$(s^\beta I + \tilde{F}(s))\tilde{Y}(s) = s^{\beta-1}\tilde{y}(0), \tag{3.12}$$

where

$$\tilde{F}(s) = \tag{3.13}$$
$$\left(\begin{array}{ccc|c|ccc}
0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & -1 & \cdots & 0 \\
 & \vdots & & \vdots & & \vdots & \\
0 & \cdots & 0 & 0 & 0 & \cdots & -1 \\
\sum_{j=1}^{n} a_{1,j} & \cdots & -a_{1,n}e^{-s\tau_{1,n}} & c\cdot\sum_{j=1}^{n} a_{1,j} & -c\cdot a_{1,2}e^{-s\tau_{1,2}} & \cdots & -c\cdot a_{1,n}e^{-s\tau_{1,n}} \\
-a_{2,1}e^{-s\tau_{2,1}} & \cdots & -a_{2,n}e^{-s\tau_{2,n}} & -c\cdot a_{2,1}e^{-s\tau_{2,1}} & c\cdot\sum_{j=1}^{n} a_{2,j} & \cdots & -c\cdot a_{2,n}e^{-s\tau_{2,n}} \\
 & \vdots & & \vdots & & \vdots & \\
-a_{n,1}e^{-s\tau_{n,1}} & \cdots & \sum_{j=1}^{n} a_{n,j} & -c\cdot a_{n,1}e^{-s\tau_{n,1}} & -c\cdot a_{n,2}e^{-s\tau_{n,2}} & \cdots & c\cdot\sum_{j=1}^{n} a_{n,j}
\end{array}\right).$$

$\tilde{Y}(s) = (\tilde{Y}_1(s), \cdots, \tilde{Y}_n(s))^T$ is the Laplace transform of $\tilde{y}(t) = (\tilde{y}_1(s), \cdots, \tilde{y}_n(s))^T$ with $\tilde{Y}_i(s) = L(\tilde{y}_i(t))$, $\tilde{y}_i(0)$ being the initial values of $\tilde{y}_i(t), i = 1, 2, \ldots, 2n$. According to the results in (Cao & Ren 2010a), the above value $b = p^T\tilde{Y}(0) + \frac{t^\beta}{\Gamma(1+\beta)}p^T\tilde{Y}^\beta(0)$, where $\tilde{Y}(0)$ and $\tilde{Y}^{(\beta)}(0)$ are the initial value of the the Laplace transform of $\tilde{y}(t)$ and $\tilde{y}^{(\beta)}(t)$, respectively.

Define the characteristic matrix of Eq. (3.11) as $\Delta(s) = s^\beta I + \tilde{F}(s)$, which is given in Theorem 3.1. It then follows that the corresponding characteristic equation can be written as follows

$$det(s^\beta I + \tilde{F}(s))) = 0. \tag{3.14}$$

In the following part, we use the same analysis method in Theorem 2.5, we obtain that if all the roots of the characteristic equation are on the left half plane or $s = 0$, then the zero solution of Eq. (3.10) is asymptotically stable, i.e.,

$$\lim_{t\to+\infty} \tilde{y}_i(t) = 0, i = 1, 2, \cdots, 2n. \tag{3.15}$$

then, we have the following result

$$\lim_{t\to+\infty} (x_i(t) - x_j(t)) = \delta_{ij}. \tag{3.16}$$

Hence, the definition of the formation producing is satisfied, the proof of this theorem is completed. ■

For the above theorem on formation producing with relative damping, we can give the same remarks as Remark 2.6, Remark 2.7, Remark 2.8, Remark 2.9 and Remark 2.10 respectively.

3.5 Simulations

In this section, several simulations are presented to illustrate the effectiveness of the control law proposed in this chapter.

We first consider a directed communication graph Fig. 3.1 with three agents in two dimensional space, it includes a spanning tree. Choose $c = 1$, $\beta = 0.9$, $\tau = 1$, and $\delta_i = (0,0)^T, i = 1, \cdots, 3$, the initial states of the three agents are chosen as $(x_1(0), y_1(0))^T = (2,-2)^T$, $(x_2(0), y_2(0))^T = (-3,3)^T$, $(x_3(0), y_3(0))^T = (-0.5, 0.5)^T$, $(x_1^{(\alpha/2)}(0), y_1^{(\alpha/2)}(0))^T = (5,3)^T$, $(x_2^{(\alpha/2)}(0), y_2^{(\alpha/2)}(0))^T = (3,1)^T$, $(x_3^{(\alpha/2)}(0), y_3^{(\alpha/2)}(0))^T = (2,2)^T$. According to Theorem 3.1, the characteristic equation of the multi-agent system with 3 agents can be calculated as follows

$$det\left(\Delta(s)\right) = \tag{3.17}$$

$$det\begin{pmatrix} s^\beta & 0 & 0 & -1 & 0 & 0 \\ 0 & s^\beta & 0 & 0 & -1 & 0 \\ 0 & 0 & s^\beta & 0 & 0 & -1 \\ 1 & 0 & -e^{-s\tau} & c+s^\beta & 0 & -c\cdot e^{-s\tau} \\ -e^{-s\tau} & 1 & 0 & -c\cdot e^{-s\tau} & c+s^\beta & 0 \\ -e^{-s\tau} & 0 & 1 & -c\cdot e^{-s\tau} & 0 & c+s^\beta \end{pmatrix} = 0. \tag{3.18}$$

which can be rewritten as the following form

$$\begin{aligned} &1 + 3s^{(\beta)} + 6s^{(2\beta)} + 7s^{(3\beta)} + 6s^{(4\beta)} + 3s^{(5\beta)} + s^{(6\beta)} - e^{(-2s)} \\ &-3s^{(\beta)}e^{(-2s)} - 4s^{(2\beta)}e^{(-2s)} - 3s^{(3\beta)}e^{(-2s)} - s^{(4\beta)}e^{(-2s)} = 0. \end{aligned} \tag{3.19}$$

For solving the nonlinear equation (3.19), we use a numerical method from Powell (1970). Let

$$\begin{cases} L_1 = 1 + 3s^{(\beta)} + 6s^{(2\beta)} + 7s^{(3\beta)} + 6s^{(4\beta)} + 3s^{(5\beta)} + s^{(6\beta)}, \\ L_2 = -e^{(-2s)} - 3s^{(\beta)}e^{(-2s)} - 4s^{(2\beta)}e^{(-2s)} - 3s^{(3\beta)}e^{(-2s)} - s^{(4\beta)}e^{(-2s)}. \end{cases} \tag{3.20}$$

From Fig. 3.2, all roots of s are located in the interval $(-5, 30)$, we search the solutions of Eq 2.31 in $(-5, 30)$. Fig. 3.3 shows the distribution for the real parts of s, we can see that all the real parts of the solutions are not more than 0 when the error is in the accepted range, which means that the real parts of s are negative or equate to zero, the conditions in the theorem are satisfied. Fig. 3.4 shows the x-state responses of the three agents under the control law (3.5), and Fig. 3.5 shows the y-state responses of three agents under the control law (3.5). Fig. 3.6 shows position trajectories of the three agents. When $\delta_i \neq (0,0)^T, i = 1,2,3$, the desired form is a triangle as Fig. 3.7, the formation producing as shown in Fig. 3.8 can also be achieved using control law 3.5.

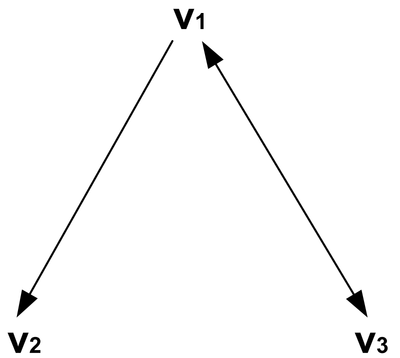


Figure 3.1: Directed communication graph of three agents.

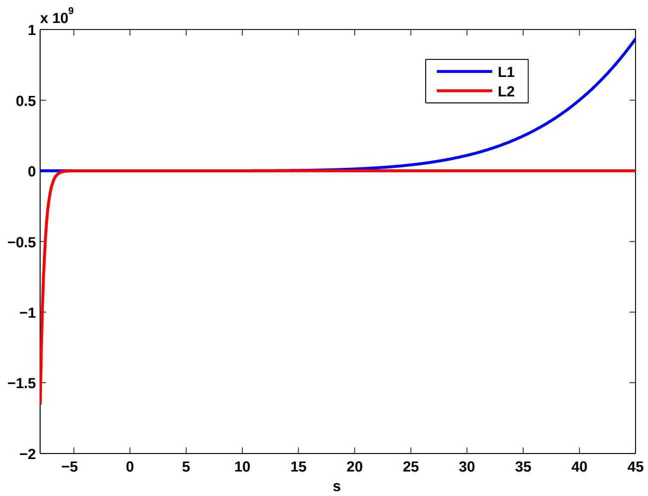


Figure 3.2: Curve diagram of L_1 and L_2 based on Eq. (3.19) for the case of three agents.

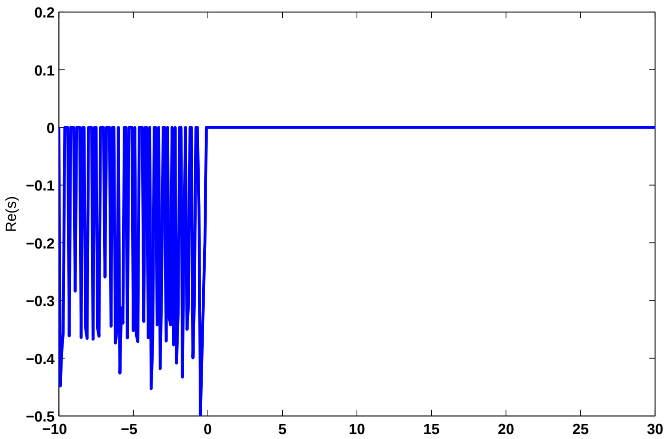


Figure 3.3: The distribution for the real parts of the roots of Eq. (3.19).

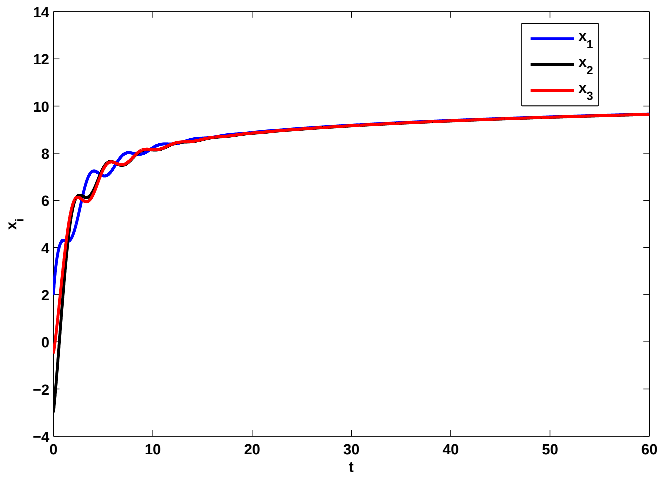


Figure 3.4: X-state responses of three agents under control law (3.5).

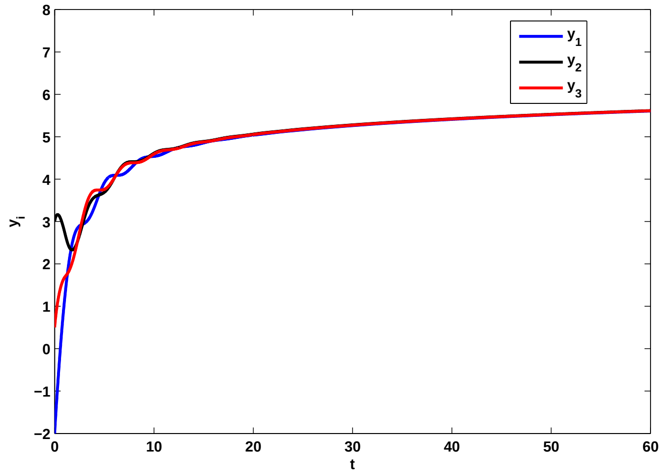


Figure 3.5: Y-state responses of three agents under control law (3.5).

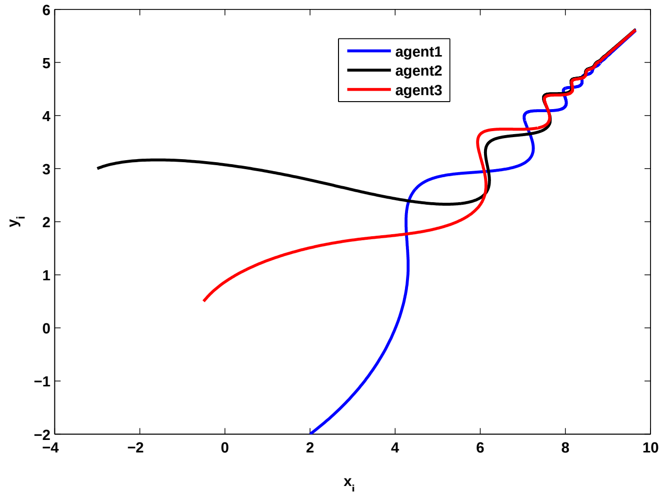


Figure 3.6: Position trajectories of three agents.

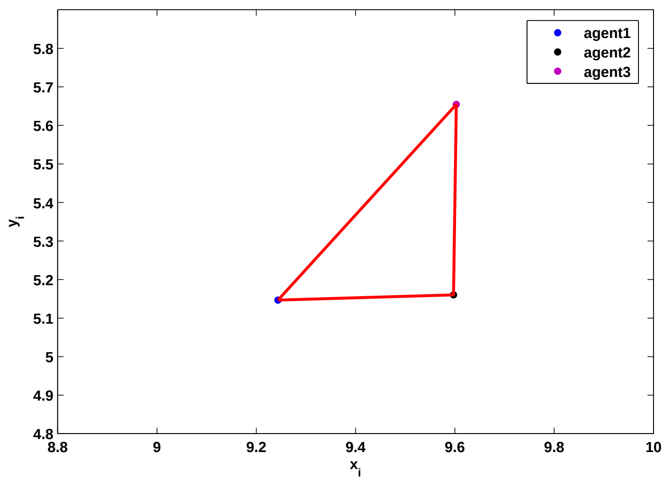


Figure 3.7: The desired formation geometric form for three agents.

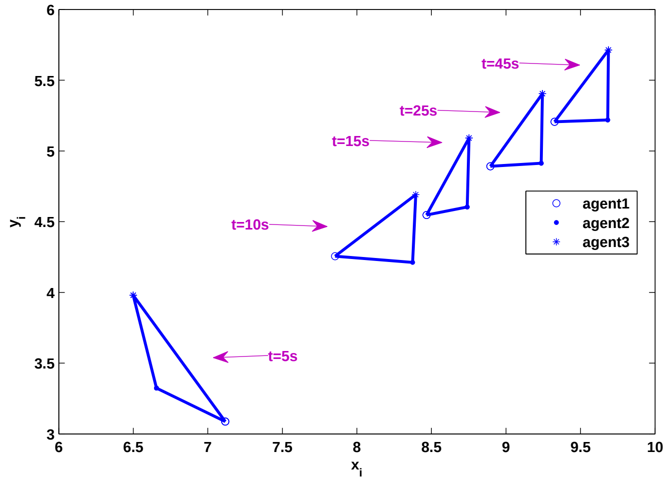


Figure 3.8: Position formation trajectories of three agents.

Next, we consider a directed communication graph with four agents in Fig. 3.9, which includes a directed spanning tree. According to the Theorem 3.1,

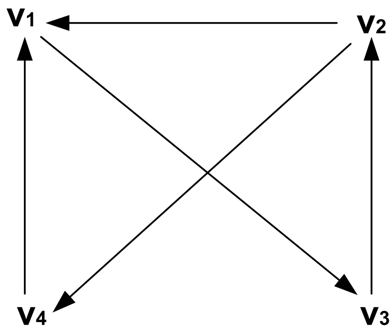


Figure 3.9: Directed communication graph of four agents.

we can calculate the characteristic equation of the multi-agent system shown as follows

$$
\begin{aligned}
&det\left(\Delta(s)\right) = \qquad\qquad (3.21)\\
&det\left(\begin{array}{cccc|c|c|c|c}
s^{\beta} & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & s^{\beta} & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & s^{\beta} & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & s^{\beta} & 0 & 0 & 0 & -1\\
2 & -e^{-s\tau} & 0 & -e^{-s\tau} & 2c+s^{\beta} & -c\cdot e^{-s\tau} & 0 & -c\cdot e^{-s\tau}\\
0 & 1 & -e^{-s\tau} & 0 & 0 & c+s^{\beta} & -c\cdot e^{-s\tau} & 0\\
-e^{-s\tau} & 0 & 1 & 0 & -c\cdot e^{-s\tau} & 0 & c+s^{\beta} & 0\\
0 & -e^{-s\tau} & 0 & 1 & 0 & -c\cdot e^{-s\tau} & 0 & c+s^{\beta}
\end{array}\right) = 0.
\end{aligned}
$$

Choose $c = 1$, $\beta = 0.95$, $\tau = 0.1$, and $\delta_i = (0, 0)^T, i = 1, \cdots, 4$, the initial states of four agents in the two dimensional space are chosen as $(x_1(0), y_1(0))^T = (3, -1)^T$, $(x_2(0), y_2(0))^T = (-2, 2)^T$, $(x_3(0), y_3(0))^T = (-2, -2)^T$,$(x_4(0), y_4(0))^T = (-1, 0)^T$, $(x_1^{(\alpha/2)}(0), y_1^{(\alpha/2)}(0))^T = (3, 0)^T$, $(x_2^{(\alpha/2)}(0), y_2^{(\alpha/2)}(0))^T = (1, -1)^T$, $(x_3^{(\alpha/2)}(0), y_3^{(\alpha/2)}(0))^T = (-1, 2)^T$, $(x_4^{(\alpha/2)}(0), y_4^{(\alpha/2)}(0))^T = (2, 3)^T$. To calculate the real parts of s, the same numerical method in (Powell (1970)) is used.

From Fig. 3.10, all roots of s are located in the interval $(-25, 20)$, we search the solutions of Eq 2.31 in $(-25, 20)$. Fig. 3.11 shows the distribution for the real parts of s, we can see that all the real parts of the solutions are not more than 0 when the error is in the accepted range, which means that the real parts of s are negative or equate to zero. Hence, the conditions in Theorem 3.1 are satisfied. Fig. 3.12 shows the x-state responses of four agents, and Fig. 3.13 shows the

y-state responses of four agents. Fig. 3.14 shows the position trajectories of the four agents under control law (3.5). When $\delta_i \neq (0,0)^T, i = 1, \cdots, 4$, the desired form is a quadrangle as Fig. 3.15, the formation producing as shown in Fig. 3.16 can also be achieved using control law 3.5.

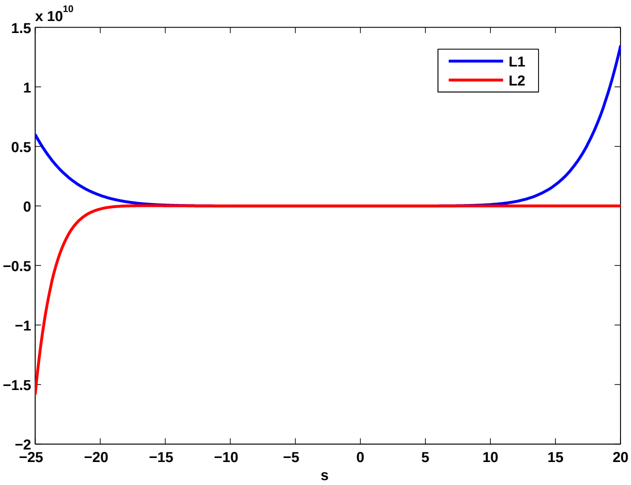


Figure 3.10: Curve diagram of L_1 and L_2 based on Eq. (3.21) for the case of four agents.

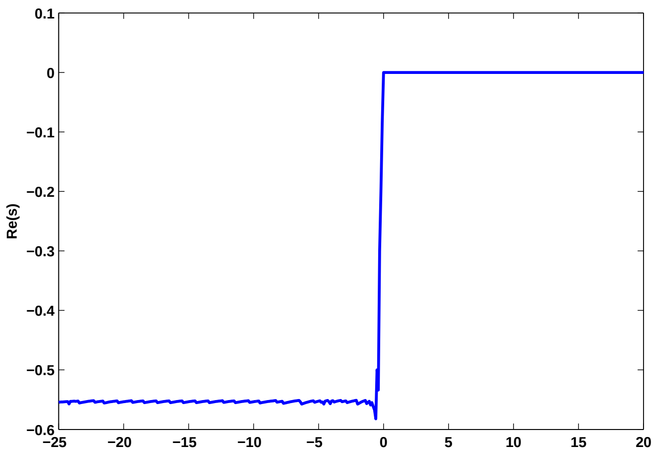


Figure 3.11: The distribution for the real parts of the roots of Eq. (3.21).

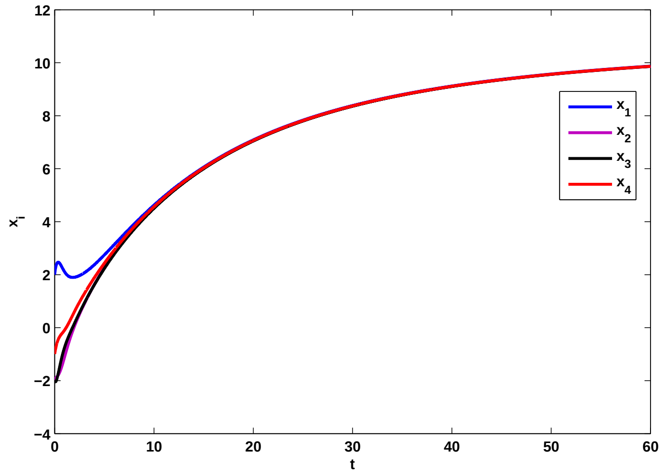


Figure 3.12: X-state responses of four agents under control law (3.5).

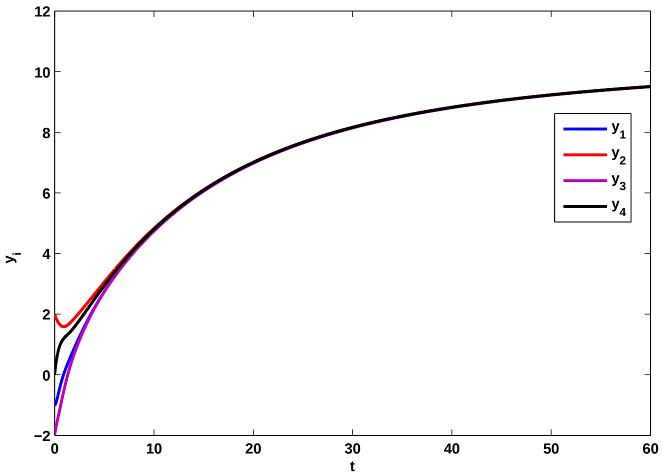


Figure 3.13: Y-state responses of four agents under control law (3.5).

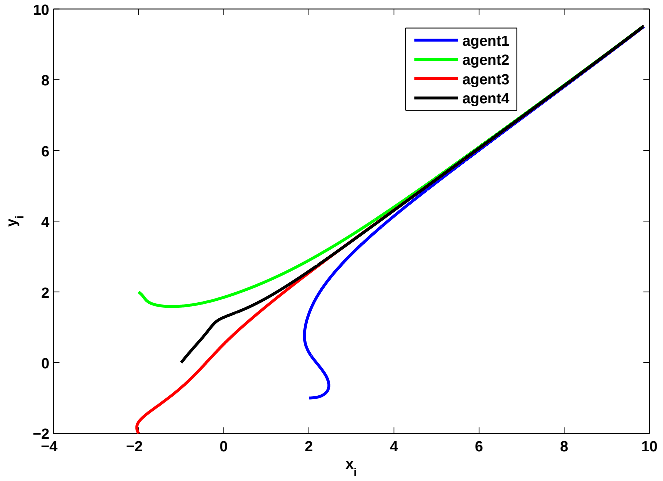


Figure 3.14: Position trajectories of four agents.

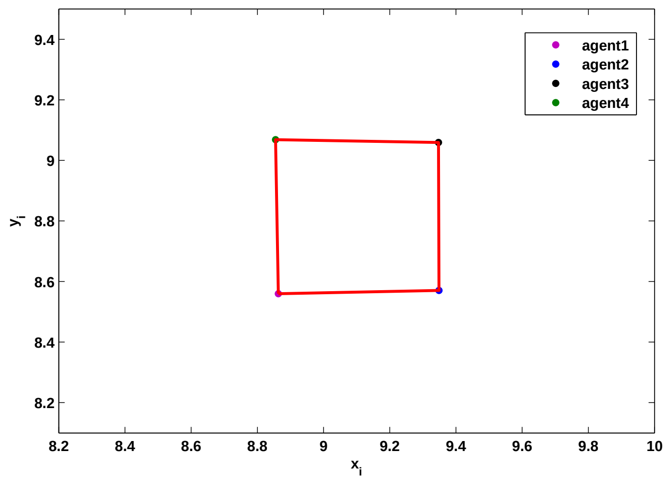


Figure 3.15: The desired formation geometric form for four agents.

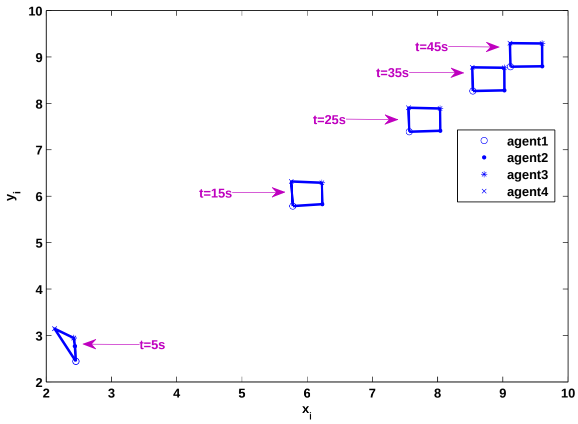


Figure 3.16: Position formation trajectories of four agents.

Finally, let's consider a group of six agents indexed by 1, 2, ..., 6, respectively. The directed communication graph Fig. 3.17 has a directed spanning tree. According to the Theorem 3.1, the characteristic equation of the multi-agent system with six agents can be calculated as follows

$$det\Big(\Delta(s)\Big) = det \tag{3.22}$$

$$\left(\begin{array}{cccccc|c|c|c|c|c|c}
s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & s^\beta & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & -e^{-s\tau} & 0 & 0 & 0 & 0 & c+s^\beta & -c\cdot e^{-s\tau} & 0 & 0 & 0 & 0 \\
0 & 1 & -e^{-s\tau} & 0 & 0 & 0 & 0 & c+s^\beta & -c\cdot e^{-s\tau} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -e^{-s\tau} & 0 & 0 & 0 & c+s^\beta & 0 & -c\cdot e^{-s\tau} & 0 \\
0 & 0 & -e^{-s\tau} & 1 & 0 & 0 & 0 & 0 & -c\cdot e^{-s\tau} & c+s^\beta & 0 & 0 \\
-e^{-s\tau} & 0 & 0 & 0 & 1 & 0 & -c\cdot e^{-s\tau} & 0 & 0 & 0 & c+s^\beta & 0 \\
0 & 0 & 0 & 0 & -e^{-s\tau} & 1 & 0 & 0 & 0 & 0 & -c\cdot e^{-s\tau} & c+s^\beta
\end{array}\right)$$

$=0.$

For solving the nonlinear equation, the same numerical method in (Powell (1970)) is used.

From Fig. 3.18, all roots of s are located in the interval $(-10, 10)$, we search the solutions of Eq 2.31 in $(-10, 10)$. Fig. 3.19 shows the distribution for the real parts of s, we can see that all the real parts of the solutions are not more than 0 when the error is in the accepted range, which means that the real parts of s are negative or equate to zero. Hence, the conditions in Theorem 3.1 are satisfied. The initial states of the six agents in the two dimensional space are chosen as $(x_1(0), y_1(0))^T = (3.5, 3)^T$, $(x_2(0), y_2(0))^T = (2, -4)^T$, $(x_3(0), y_3(0))^T = (2, -2)^T$,$(x_4(0), y_4(0))^T = (-1, 0)^T$, $(x_5(0), y_5(0))^T = (-1, 1)^T$,$(x_6(0), y_6(0))^T = (0.5, 1)^T$, $(x_1^{(\alpha/2)}(0), y_1^{(\alpha/2)}(0))^T = (1, 0)^T$, $(x_2^{(\alpha/2)}(0), y_2^{(\alpha/2)}(0))^T = (-1, 1)^T$, $(x_3^{(\alpha/2)}(0), y_3^{(\alpha/2)}(0))^T = (2, 2)^T$, $(x_4^{(\alpha/2)}(0), y_4^{(\alpha/2)}(0))^T = (3, 2)^T$, $(x_5^{(\alpha/2)}(0), y_5^{(\alpha/2)}(0))^T = (1, 0)^T$, $(x_6^{(\alpha/2)}(0), y_6^{(\alpha/2)}(0))^T = (-1, 1)^T$, and $c = 1$, $\beta = 0.9$, $\tau = 0.1$, $\delta_i = (0, 0)^T, i = 1, \cdots, 6$ are chosen. Fig. 3.20 shows the x-state responses of six agents, and Fig. 3.21 shows the y-state responses of six agents. Fig. 3.24 shows the position trajectories of the six agents under control law (3.5). When $\delta_i \neq (0, 0)^T, i = 1, \cdots, 6$, the desired form is an hexagon as Fig 3.23, the formation producing as shown in Fig. 3.24 can also be achieved using control law 3.5.

Above simulations verify that consensus producing can be viewed as a part of formation producing and the above theories on formation producing are effective.

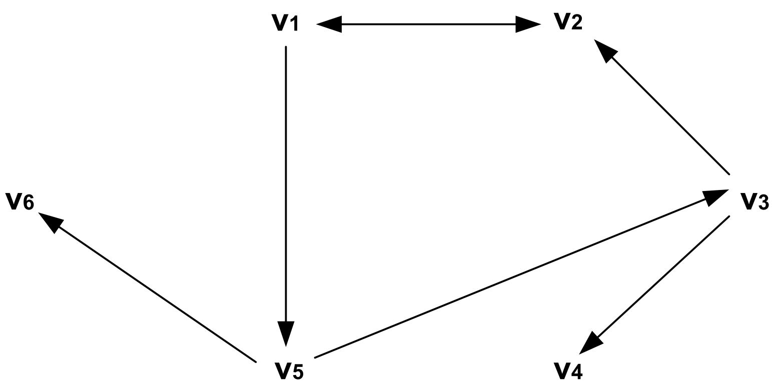


Figure 3.17: Directed communication graph of six agents.

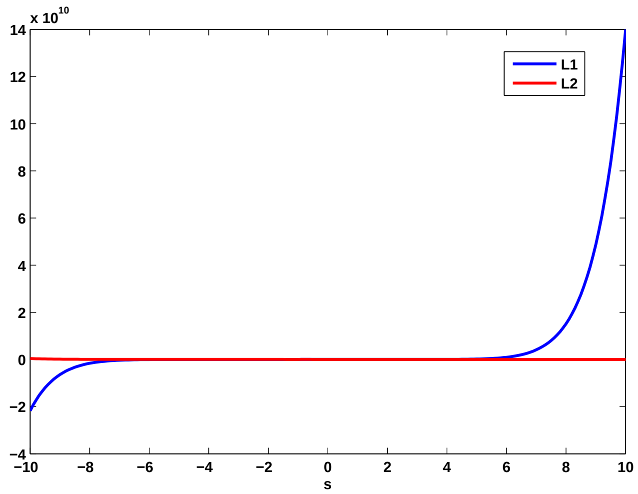


Figure 3.18: Curve diagram of L_1 and L_2 based on Eq. (3.22) for the case of six agents.

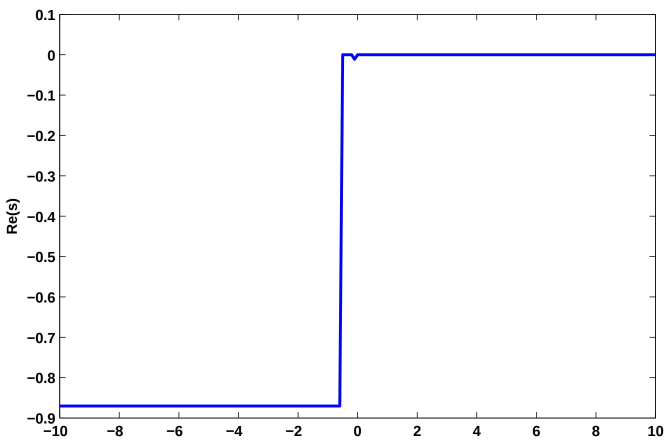


Figure 3.19: The distribution for the real parts of the roots of Eq. (3.22).

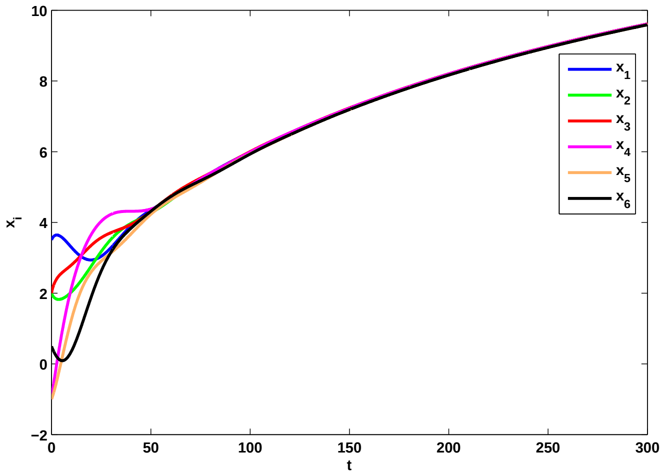


Figure 3.20: X-state responses of six agents under control law (3.5).

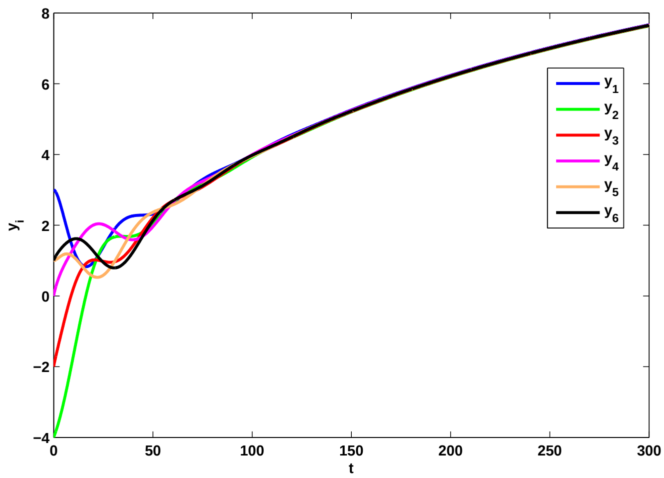


Figure 3.21: Y-state responses of six agents under control law (3.5).

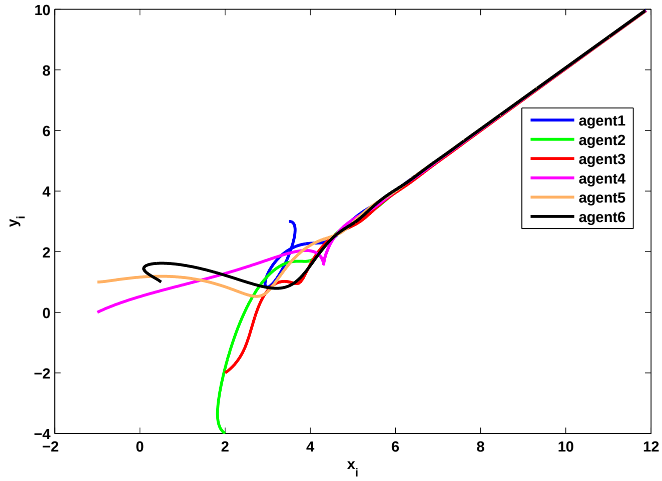


Figure 3.22: Position trajectories of six agents.

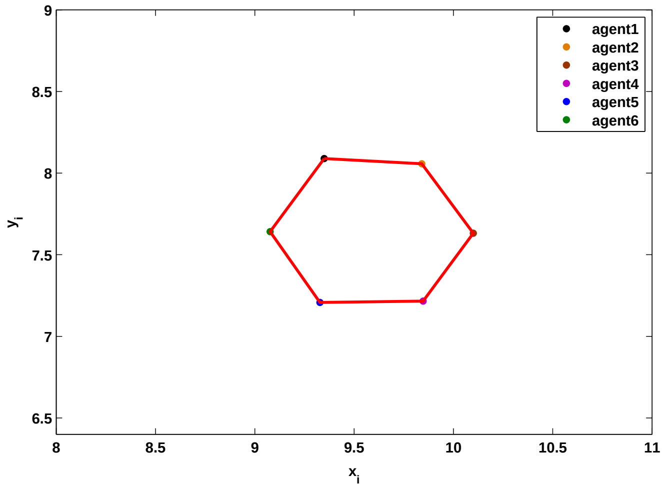


Figure 3.23: The desired formation geometric form for six agents.

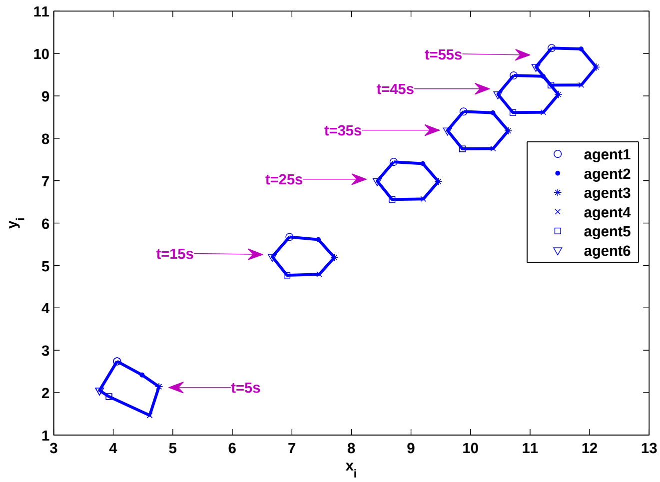


Figure 3.24: Position formation trajectories of six agents.

3.6 Conclusion

In this chapter we have studied the distributed formation producing problem for fractional-order multi-agent systems with relative damping and communication delay. Firstly, fractional-order multi-agent systems and a control law have been provided, by applying vector conversion, the nonlinear systems are changed into linear systems. Then, using the matrix theory, graph theory and the frequency domain analysis, the conditions of formation producing have been given under the form of a theorem. Finally, the simulations have been given to verify the validity of our theoretical analysis. Consensus/formation tracking of fractional-order systems is one of the most interesting topics in our research work on multi-agent systems, it will be studied in the next chapters.

Chapter 4

Consensus/Formation Tracking of Fractional-Order Multi-Agent Systems Based on Error Predictor

Contents

4.1 Introduction

Note that chapter 2 and chapter 3 studied formation control without a reference state, the final target value to be reached is an inherent point or trajectory. However, it is desirable that the states of all agents can asymptotically track a reference state, which can be any constant point or a time-varying state. A reference state represents the state of common interest for all other agents, which is required in many practical applications, examples include formation flying, body guard, and coordinated tracking applications.

Consensus means that a group of agents reach an agreement on a comment feature in the presence of a reference state. Presently many publications on consensus tracking are available. For example, in (Jadbabaie *et al.* 2003; Moore & Lucarelli 2007), the consensus problem with a constant reference state was addressed respectively under a switching network topology and a fixed directed network topology. Then, the consensus with a time-varying reference state case was also studied (Peng *et al.* 2014; Ren 2007). Consensus control laws (Cao *et al.* 2009; Ren 2007) were proposed and studied for single-integrator kinematics in the presence of a reference state in both continuous-time and discrete-time settings. Multiple leaders case(Ji *et al.* 2008), using a stop and go strategy to drive agents to the convex polytope spanned by leaders was proposed. Furthermore, Hong *et al.* 2006 solved the consensus problem with a time-varying leader under an undirected network topology, with the condition that the acceleration of the leader was available to each all agents.

Formation tracking means controlling a group of agents such that desired formation shapes and cooperative tasks can be achieved in the presence of a reference state. We have introduced in Chapter 1 that consensus problem is considered as a part of formation control problem, which means that the latter results can be used in the consensus problem. But formation tracking demands both a reference state and desired formation geometrics. Hence, the results in consensus need to be extended to formation tracking problem. Many results on formation tracking were also obtained. For example, the matrix approach and Lyapunov approach were also used in formation tracking (Cao & Ren 2012; Wang *et al.* 2010a; Wen *et al.* 2012a). In addition, Do 2008 investigated a constructive method to design cooperative controllers, which can force a group of unicycle-type mobile robots with limited sensing ranges to perform desired formation tracking. Moreover, Fang & Antsaklis 2006 considered formation tracking of nonlinear multi-vehicle dynamics. Time delays and noise disturbance were considered for formation tracking (Lai *et al.* 2014). However, there are few results on consensus of fractional-order with a reference state (Zhao *et al.* 2012).

Comparing with existing results, this chapter has the following differences. Firstly, in contrast to the studies without a reference state (Dong 2012; Lin *et al.*

2004; Rao & Ghose 2011), this chapter considers the consensus of multi-agent systems with a reference state. Secondly, different from the results on coordination of integer-order multi-agent systems (Guoguang Wen & Yu 2011; Hong *et al.* 2006; Ji *et al.* 2008; Peng & Yang 2009), in this chapter, the consensus problem of multi-agent systems is studied based on fractional-order systems. Two types of effective control laws are given. Finally, the convergence speed is compared based on the proposed two types of control laws. In this chapter, we shall investigate consensus/formation tracking of fractional-order multi-agent systems with a reference state, it is organized as follows: Firstly, the common control law is proposed, and validated when the communication graph has a directed spanning tree. Secondly, the control law based on error predictor is proposed, and validated when the communication graph has a directed spanning tree. Then the convergence speeds of fractional-order multi-agent systems based on the above control laws are compared. It is verified that the convergence of systems is faster using the control law based on error predictor than using the common ones. Thirdly, the control law based on error predictor is extended to solve the formation tracking problem. Finally, several simulations are presented to verify the validity of the obtained results.

4.2 Preliminaries

In this chapter, the reference state x_0 is represented by vertex v_0, which has been defined in subsection 1.1.2.1. Then, we have a fixed communication graph $\bar{G}$, which consists of communication graph G in chapter 1, vertex v_0 and edges between a reference state and its neighbors. The reference state is independent and it gives its state information to its neighbors. The motion of each agent is influenced by the reference state and its neighbors.

Definition 4.1 *For $\bar{G}$, we say that the node v_0 is globally reachable, if there is a path in $\bar{G}$ from the node v_0 to every node v_i in $\bar{G}$.*

The next lemma shows an important property of Laplace Matrix L (Lin *et al.* 2005),

Lemma 4.2 *A fixed communication graph G has a globally reachable point, if and only if the Laplace matrix L of G has a simple zero eigenvalue with $\mathbf{1} = (1, 1, \cdots, 1)^T \in R^n$.*

From Lemma 1.6 and Lemma 4.2, the following Lemma can be given

Lemma 4.3 *A fixed communication graph G has a globally reachable node, if and only if the communication graph has a directed spanning tree.*

4.3 Problem Description

In this chapter, a simple notation $x^{(\alpha)}(t)$ is used to denote the Caputo fractional operator as that in Chapter 1. The systems of agent i ($i \in N : N = \{1, \cdots, n\}$) is described as follows

$$x_i^{(\alpha)}(t) = u_i(t), \quad i = 0, 1, \cdots, n \tag{4.1}$$

where $\alpha \in (0, 1]$, $x_i(t) \in R$ and $u_i(t) \in R$ represent the state and the control input of system. $x_i^{(\alpha)}(t)$ is the αth Caputo derivative of $x_i(t)$.

Definition 4.4 *For any initial condition $x_i(0)$, $i = 0, 1, \cdots, n$, the consensus problem with a reference state can be solved using control laws if the states of agent i asymptotically approach the reference state $x_0(t)$, as $t \to \infty$. That is*

$$\lim_{t \to +\infty} (x_i(t) - x_0(t)) = 0, \quad i \in N \tag{4.2}$$

4.4 Consensus with a Reference State

In this section, the case of fixed communication graph is considered. We design control laws such that all agents can track the reference state with local interaction. Firstly, the common control law is given to solve the consensus problem. Then, the control law based on error predictor is also proposed to improve the convergence speed.

4.4.1 Consensus with a common control law

In this subsection, the common control law is proposed

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t) - x_i(t)] + a_{i,0}[x_0(t) - x_i(t)] + u_0(t), i, j \in N, i \neq j, \alpha \in (0, 1] \tag{4.3}$$

where $a_{i,j}, i, j \in N$ is the $(i, j)th$ entry of the adjacency matrix A, $a_{i,0}$ is a positive constant if the reference state is available to agent i and $a_{i,0} = 0$ otherwise.

To study the consensus problem with a reference state, we define a diagonal matrix $B \in R^{n\times n}$ to be a reference state adjacency matrix associated diagonal elements $b_i, i \in N$, where $b_i = a_{i,0} > 0$, if the reference state is a neighbor of agent i and $b_i = a_{i,0} = 0$, otherwise.

Example 4.5 *As shown in Fig. 4.2 and Fig. 4.10, $\bar{G}_1$, $\bar{G}_2$ and have a globally reachable node v_0.*

The Laplace matrices L_1 and L_2 without node v_0 as well as the reference state adjacency matrices B_1 and B_2 are easily obtained as follows

$$L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, L_2 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{4.4}$$

$$B_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.5}$$

Let $M = L + B$, which plays a key role in the convergence analysis of the error systems. The following Definition and Lemma show a relationship between M

and the directed graph $\bar{G}$.

Definition 4.6 *M is called a positive stable matrix, if all eigenvalues of M have positive real parts.*

Lemma 4.7 *(Hu & Hong 2007) The matrix $M = L + B$ is positive stable, if and only if node v_0 is globally reachable.*

The following lemma is given, which will play an important role in the proof of the main results.

Lemma 4.8 *(Matignon 1996). The following autonomous system:*

$$x^{(\alpha)}(t) = Ax(t), \quad x(0) = x_0, \alpha \in (0, 1] \tag{4.6}$$

with $x \in R^n$, and $A \in R^{n \times n}$, is asymptotically stable if and only if $|arg(\lambda(A))| > \alpha\pi/2$ is satisfied for all eigenvalues of matrix A. Also, this system is stable if and only if $|arg(\lambda(A))| \geqslant \alpha\pi/2$ is satisfied for all eigenvalues of matrix A with those critical eigenvalues satisfying $|arg(\lambda(A))| = \alpha\pi/2$ having geometric multiplicity of one. The geometric multiplicity of an eigenvalue λ of matrix A is the dimension of subspace of vector v for which $Av = \lambda v$.

The stability domain can be expressed as follows

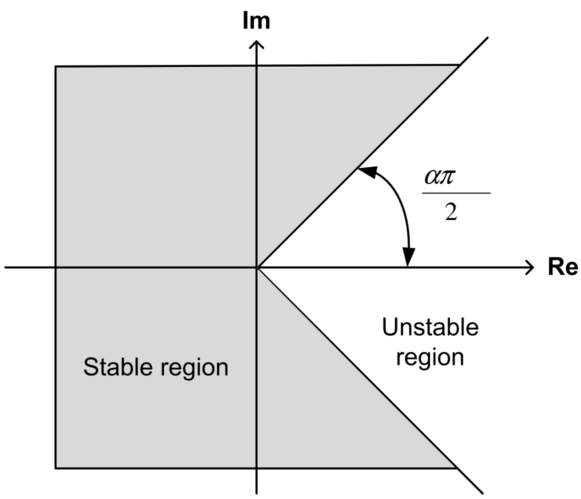


Figure 4.1: Stability domain for linear fractional-order systems with $\alpha \in (0, 1]$.

Theorem 4.9 *Control law (4.3) solve the consensus problem with a reference state $x_0(t)$, if directed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. Substituting (4.3) into (4.1), n systems (4.1) can be written as the following compact form

$$\begin{aligned} x^{(\alpha)}(t) &= -Mx(t) + B(x_0(t)\mathbf{1}) + u_0(t) \\ &= -Mx(t) + B(x_0(t)\mathbf{1}) + x_0^{(\alpha)}(t)\mathbf{1}, \end{aligned} \tag{4.7}$$

where $M = L + B$, $x(t) = (x_1(t), \cdots, x_n(t))^T$.

Noting that $\tilde{x}_i(t) = x_i(t) - x_0(t)$, we only need to consider the following system errors

$$\begin{aligned} \tilde{x}^{(\alpha)}(t) &= x^{(\alpha)}(t) - x_0^{(\alpha)}(t)\mathbf{1} \\ &= -Mx(t) + B(x_0(t)\mathbf{1}) + x_0^{(\alpha)}(t)\mathbf{1} - x_0^{(\alpha)}(t)\mathbf{1} \\ &= -(L + B)x(t) + B(x_0(t)\mathbf{1}) \\ &= -L(x(t) - x_0(t)\mathbf{1}) - B(x(t) - x_0(t)\mathbf{1}) \\ &= -M\tilde{x}(t), \end{aligned} \tag{4.8}$$

where $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (x_1(t) - x_0(t), \cdots, x_n(t) - x_0(t))^T$, $L(x(t) - x_0(t)\mathbf{1}) = Lx(t)$ is applied.

Due to directed communication graph $\bar{G}$ has a directed spanning tree, according to the Lemma 4.3, directed communication graph $\bar{G}$ has a globally reachable note v_0, and Lemma 4.7 guarantees that M is positive stable, which means that all the real parts of the eigenvalues of matrix M are positive, according to Lemma 4.8, the system errors are asymptotically stable, such that

$$\lim_{t \to +\infty} \tilde{x}_i(t) = 0, \quad i \in N \tag{4.9}$$

That is

$$\lim_{t \to +\infty} (x_i(t) - x_0(t)) = 0. \;\; i \in N \tag{4.10}$$

Then, the consensus with a reference state is achieved by control law (4.3) if directed communication graph $\bar{G}$ has a directed spanning tree . ■

To verify the validity of the Theorem 4.9, several simulations are given below in two dimensional space. Firstly, consider directed communication graph shown in Fig. 4.2, which includes a directed spanning tree. Let $\alpha = 0.95$ and the time-varying reference state $[x_0^{(\alpha)}(t), y_0^{(\alpha)}(t)]^T = [t, sin(\pi \times t/10)]^T$. The initial conditions are chosen as $[x_0(0), y_0(0)]^T = [2, 0.5]^T$, $[x_1(0), y_1(0)]^T = [-2, -10]^T$, $[x_2(0), y_2(0)]^T = [-10, 4]^T$, and $[x_3(0), y_3(0)]^T = [-8, 12]^T$. As shown in Fig. 4.3 and Fig. 4.4, the state errors between agents i and the reference state are got along x-axis and y-axis. That is, the system errors converge to 0 after a period of time using control law (4.3). Furthermore, the position trajectories of three agents are obtained in Fig. 4.5, where the reference state is denoted by a solid line, and the states of all agents are denoted by dashed lines. It is shown that the states of all agents converge to the reference state eventually.

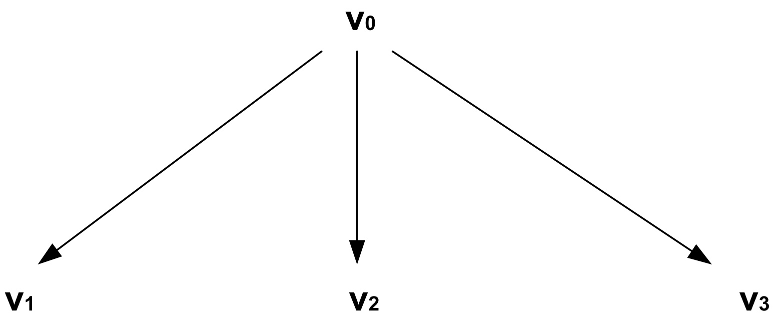


Figure 4.2: Directed communication graph for a group of three agents and a reference state.

Secondly, the directed communication graph with four agents is considered in Fig. 4.6, which includes a directed spanning tree. Let $\alpha = 0.95$ and the time-varying reference state $[x_0^{(\alpha)}(t), y_0^{(\alpha)}(t)]^T = [sin(t/10), cos(t/50)]^T$. The initial conditions are chosen as $[x_0(0), y_0(0)]^T = [2, 0.5]^T$, $[x_1(0), y_1(0)]^T = [2, -10]^T$, $[x_2(0), y_2(0)]^T = [-10, 4]^T$, $[x_3(0), y_3(0)]^T = [-8, 12]^T$, and $[x_4(0), y_4(0)]^T = [5, -5]^T$. As shown in Fig. 4.7 and Fig. 4.8, where the system errors between agents i and the reference state are achieved along x-axis and y-axis. That is, the system errors approach 0 after a period of time using control law (4.3). The position trajectories of four agents are got in Fig. 4.9, the reference state is denoted by a solid line, and the states of all agents are denoted by dashed lines. It is shown that the states of all agents converge to the reference state.

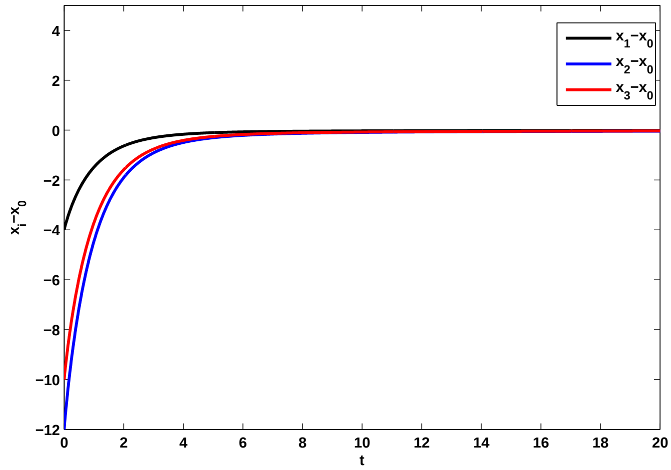


Figure 4.3: The system errors between three agents i ($i = 1, 2, 3$) and the reference state along x-axis using control law (4.3).

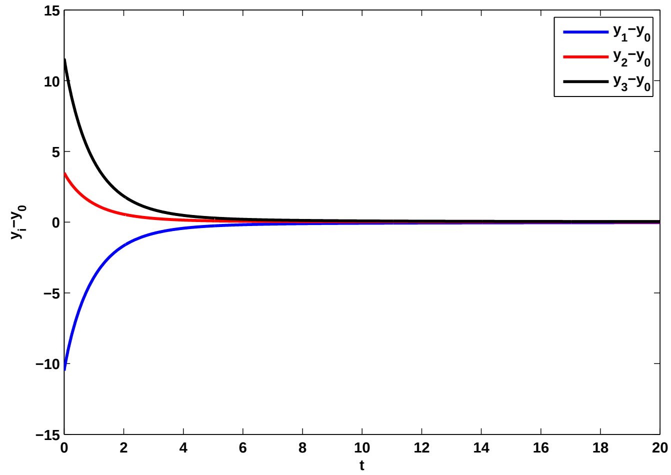


Figure 4.4: The system errors between three agents i ($i = 1, 2, 3$) and the reference state along y-axis using control law (4.3).

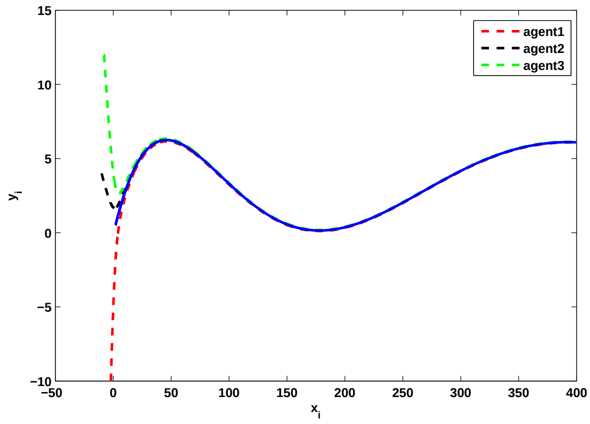


Figure 4.5: Position trajectories of three agents.

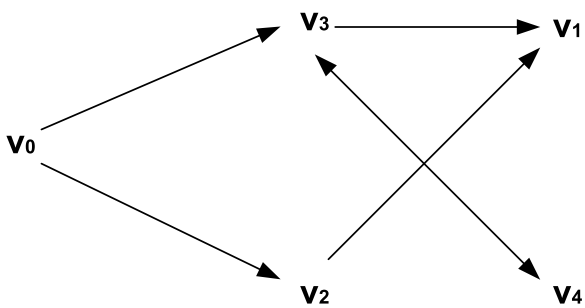


Figure 4.6: Directed communication graph for a group of four agents and a reference state.

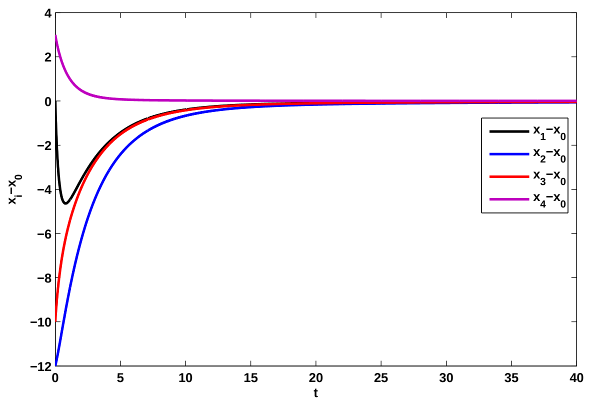


Figure 4.7: The system errors between four agents i $(i = 1, 2, 3, 4)$ and the reference state along x-axis using control law (4.3).

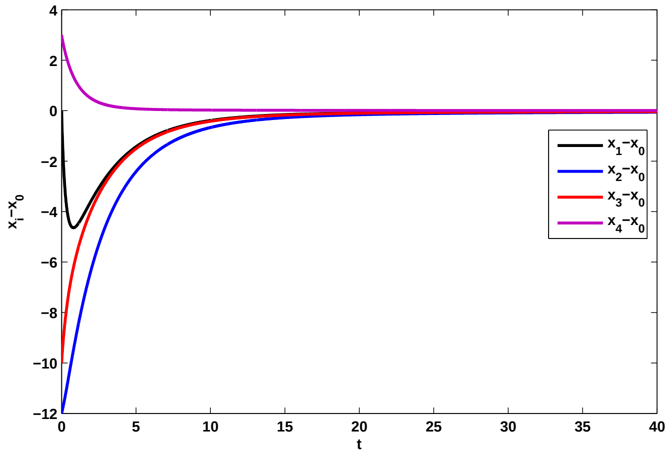


Figure 4.8: The system errors between four agents i ($i = 1, 2, 3, 4$) and the reference state along y-axis using control law (4.3).

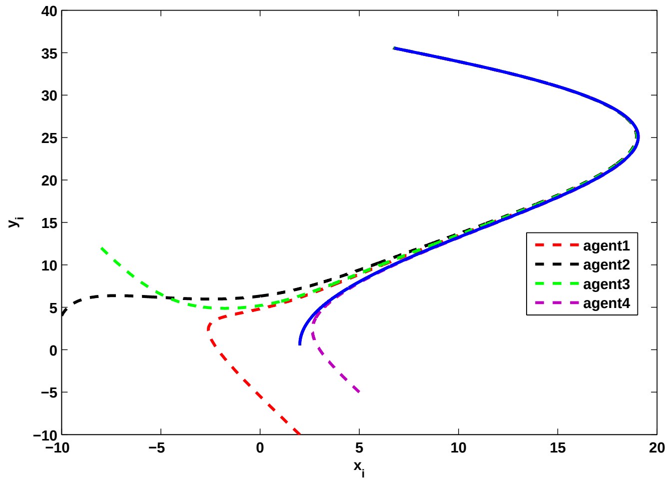


Figure 4.9: Position trajectories of four agents.

Finally, the directed communication graph with six agents is studied in Fig. 4.10, which contains a directed spanning tree. Let $\alpha = 0.95$ and the time-varying reference state $[x_0^{(\alpha)}(t), y_0^{(\alpha)}(t)]^T = [sin(t/30), cos(\pi \times t/50)]^T$. The initial conditions are given as $[x_0(0), y_0(0)]^T = [2, 3]^T$, $[x_1(0), y_1(0)]^T = [1, -1]^T$, $[x_2(0), y_2(0)]^T = [5, 2]^T$, $[x_3(0), y_3(0)]^T = [-3, 1]^T$, $[x_4(0), y_4(0)]^T = [3, 0.5]^T$, $[x_5(0), y_5(0)]^T = [-5, 2]^T$, $[x_6(0), y_6(0)]^T = [-2, -3]^T$. As shown in Fig. 4.11 and Fig. 4.12, the system errors between agents i and the reference state are obtained along x-axis and y-axis, where the system errors approach 0 after a period of time using the control law (4.3). Then, the position trajectories of six agents are achieved in Fig. 4.13, the reference state is denoted by a solid line, and the states of all agents are denoted by dashed lines. It is shown that the states of all agents converge to the reference state eventually. All the above simulations verify the effectiveness of the Theorem 4.9.

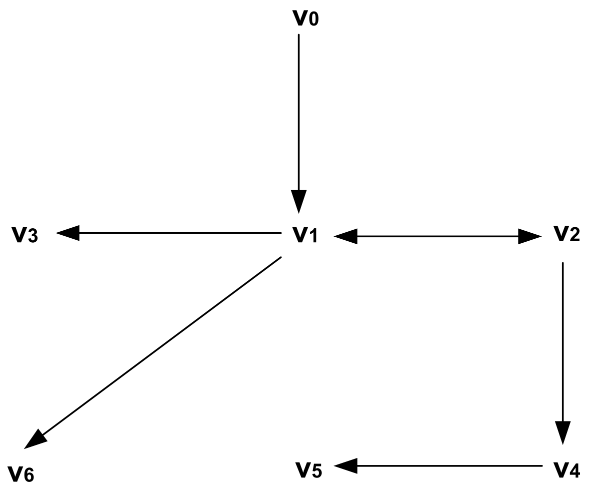


Figure 4.10: Directed communication graph for a group of six agents and a reference state.

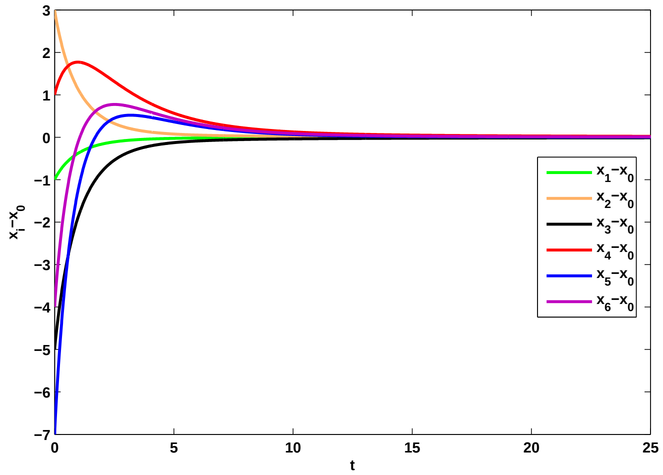


Figure 4.11: The system errors between six agents i ($i = 1, 2, 3, 4, 5, 6$) and the reference state along x-axis using control law (4.3).

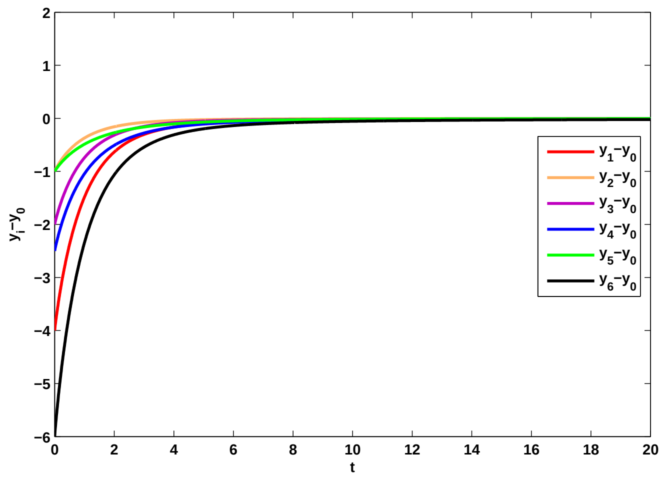


Figure 4.12: The system errors between six agents i ($i = 1, 2, 3, 4, 5, 6$) and the reference state along y-axis using control law (4.3).

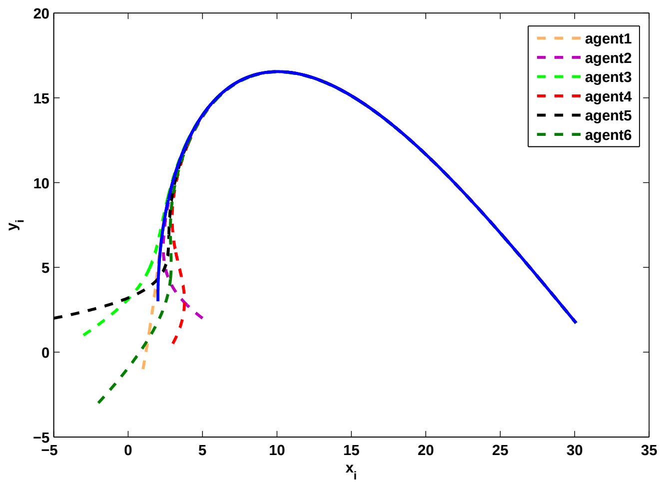


Figure 4.13: Position trajectories of six agents.

4.4.2 Consensus with a control law based on error predictor

In this subsection, the control law based on the error predictor is introduced to solve the consensus problem. It is shown that the control law can improve the convergence speed of consensus by comparing to the common control law (4.3).

Let $\tilde{x}_i(t) = x_i(t)-x_0(t)$, and $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (x_1(t)-x_0(t), \cdots, x_n(t) - x_0(t))^T$. An error predictor is introduced as follows

$$e_i(t) = \eta(\sum_{j=1}^{n} a_{i,j}[\tilde{x}_j(t) - \tilde{x}_i(t)] - a_{i,0}\tilde{x}_i(t)), \tag{4.11}$$

where $\eta > 0$ is the impact factor of the error predictor. Let $e(t) = (e_1(t), \cdots, e_n(t))^T$, then

$$e(t) = \eta(-Mx(t) + Bx_0(t)\mathbf{1}). \tag{4.12}$$

Based on the above error predictor, a new consensus control law is given as follows

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t) - x_i(t)] + a_{i,0}[x_0(t) - x_i(t)] + e_i(t) + u_0(t), \quad i, j \in N, i \neq j \tag{4.13}$$

where $a_{i,j}$ and $a_{i,0}$ are defined as control law (4.3).

Theorem 4.10 *control law (4.13) solves the consensus problem with a time-varying reference state, if directed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. Submitting control law (4.13) into the systems (4.1), we can get the following equation

$$\begin{aligned} x^{(\alpha)}(t) =& -Mx(t) + B(x_0(t)\mathbf{1}) + e(t) + u_0(t) \\ =& -Mx(t) + B(x_0(t)\mathbf{1}) + e(t) + x_0^{(\alpha)}(t)\mathbf{1}. \end{aligned} \tag{4.14}$$

Note that information feedback is transmitted to each agent through its local neighbors' state information and their derivatives. For the above systems, the following system errors can be given as follows

$$\begin{aligned} \tilde{x}^{(\alpha)}(t) =& x^{(\alpha)}(t) - x_0^{(\alpha)}(t)\mathbf{1} \\ =& -Mx(t) + Bx_0(t)\mathbf{1} + \eta(-Mx(t) + Bx_0(t)\mathbf{1}) + x_0^{(\alpha)}(t)\mathbf{1} - x_0^{(\alpha)}(t)\mathbf{1} \\ =& (I + \eta I)(-Mx(t) + Bx_0(t)\mathbf{1}) \\ =& (I + \eta I)(-(L + B)x(t) + Bx_0(t)\mathbf{1}) \\ =& (I + \eta I)(-L(x(t) - x_0(t)\mathbf{1}) - B(x(t) - x_0(t)\mathbf{1})) \\ =& -(I + \eta I)M\tilde{x}(t), \end{aligned} \tag{4.15}$$

where $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (x_1(t) - x_0(t), \cdots, x_n(t) - x_0(t))^T$, $L(\tilde{x}(t) - x_0(t)\mathbf{1}) = L\tilde{x}(t)$ is applied. Due to directed communication graph $\bar{G}$ has a directed spanning tree, according to the Lemma 4.3, directed communication graph $\bar{G}$ has a globally reachable note v_0, and Lemma 4.7 guarantees that M is positive stable, which means that all the real parts of the eigenvalues of matrix $(I + \eta I)M$ are positive, according to Lemma 4.8, the error systems are asymptotically stable, such that

$$\lim_{t \to +\infty} \tilde{x}_i(t) = 0, \quad i \in N \tag{4.16}$$

That is

$$\lim_{t \to +\infty} (x_i(t) - x_0(t)) = 0, \quad i \in N \tag{4.17}$$

Then, the consensus with a reference state is achieved by control law (4.13) if directed communication graph $\bar{G}$ has a directed spanning tree. ■

To compare the convergence speeds of consensus using the above two types of control laws, the following useful lemma is given for our demonstration.

Lemma 4.11 *When the order α is fixed, $|arg(\lambda)| > \alpha\pi/2$, the convergence speed of the autonomous system (4.6) is relative with the largest eigenvalue λ of A. λ_{max} describes the lowest bound of the convergence speed of multi-agent system, which means that if λ_{max} is smaller, then convergence of the dynamics is faster.*

Proof. According to the results (Matignon 1996), the convergence speed of consensus of fractional-order multi-agent systems is decided by the Mittag-Leffler functions as follows

$$E_{\alpha}^{j}(\lambda, t) = t^{(j-1)\alpha} \sum_{k=0}^{\infty} C_{j-1+k}^{j-1} \frac{(\lambda t^{\alpha})^{k}}{\Gamma(1 + (j-1+k)\alpha)}, \tag{4.18}$$

where λ is the eigenvalue of matrix A, α is the order of fractional-order system, j is the multiplicity of λ. By using the theorem 1 in (Matignon 1996), it is shown that when $|arg(\lambda)| > \alpha\pi/2$, the components of the state decay contain λ and order α as follows

$$E_{\alpha}^{j}(\lambda, t) \sim \frac{1}{\Gamma(1-\alpha)} (-\lambda)^{-j} t^{-\alpha}, \tag{4.19}$$

which means that when the order α is fixed, the convergence speed is decided by the largest Mittag-Leffler function. When the convergence speeds of two autonomous systems are compared, we just need to compare the largest eigenvalue λ_{max} of each matrix A, if the λ_{max} is smaller, the convergence of the dynamics is faster. ■

Theorem 4.12 *If directed communication graph $\bar{G}$ has a directed spanning tree, the consensus of fractional-order multi-agent systems using control law (4.13) based on the error predictor can be achieved faster than the one using control law (4.3).*

Proof. To study the convergence speed of consensus using control law (4.3) and (4.13), we just need to consider the error systems (4.8) and (4.15). According to the Lemma 4.11, we just need to compare the largest eigenvalues of matrix $-M$ and $-(M+\eta M)$. Obviously, the largest eigenvalue of $-(M+\eta M)$ is smaller than the largest eigenvalue of $-M$, that means the convergence of consensus using control law (4.13) is faster than the one using control law (4.3). ■

Remark 4.13 *When $\eta = 0$, control law based on error predictor turns into the common control law. That is, the latter is a special case of the control law based on error predictor.*

Simulations are presented to illustrate the effectiveness of the Theorem 4.10 and the Theorem 4.12. We consider the same directed communication graph as shown in Fig. 4.2, Fig. 4.6 and Fig. 4.10.

Firstly, for the directed communication graph with three agents, let $\alpha = 0.95$, $\eta = 2$, the time-varying reference state and the initial condition as above subsection. As shown in Fig. 4.14 and Fig. 4.15, the system errors approach 0 after a period of time along x-axis and y-axis. The position trajectories are obtained in Fig. 4.16, where the reference state is denoted by a solid line, and the states of all agents are denoted by dashed lines. It is shown that the consensus is achieved using control law (4.13) and the convergence of fractional-order multi-agent system is faster by comparing to the result with three agents in subsection 4.3.1.

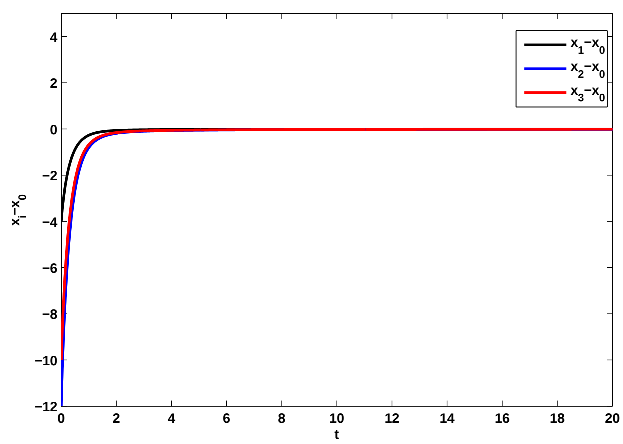


Figure 4.14: The system errors between three agents i ($i = 1, 2, 3$) and the reference state along x-axis using control law (4.13).

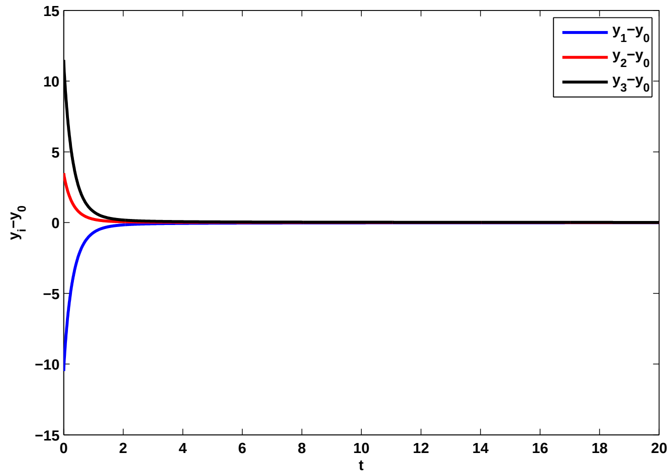


Figure 4.15: The system errors between three agents i $(i = 1, 2, 3)$ and the reference state along y-axis using control law (4.13).

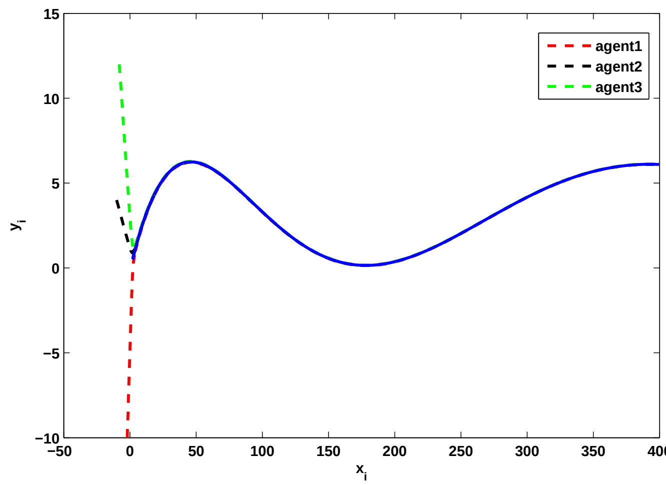


Figure 4.16: Position trajectories of three agents.

Secondly, for the directed communication graph with four agents, let $\alpha = 0.95$, $\eta = 2.5$, the time-varying reference state and the initial condition are defined as above subsection. As shown in Fig. 4.17 and Fig. 4.18, the system errors approach 0 after a period of time along x-axis and y-axis. And the position trajectories are obtained in Fig. 4.19. It is shown that the consensus is achieved using control las (4.13) and the convergence of fractional-order multi-agent system is faster by comparing to the result with four agents in subsection 4.3.1.

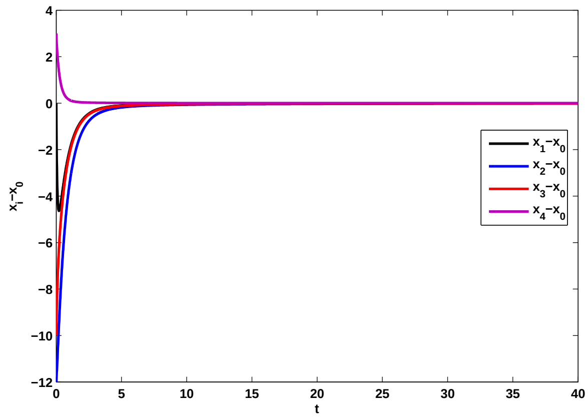


Figure 4.17: The system errors between four agents i ($i = 1, 2, 3, 4$) and the reference state along x-axis using control law (4.13).

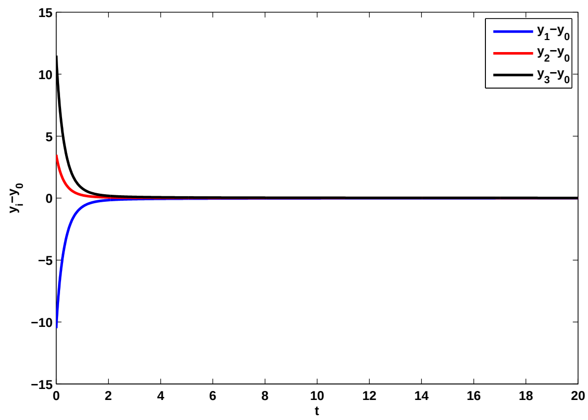


Figure 4.18: The system errors between four agents i ($i = 1, 2, 3, 4$) and the reference state along y-axis using control law (4.13).

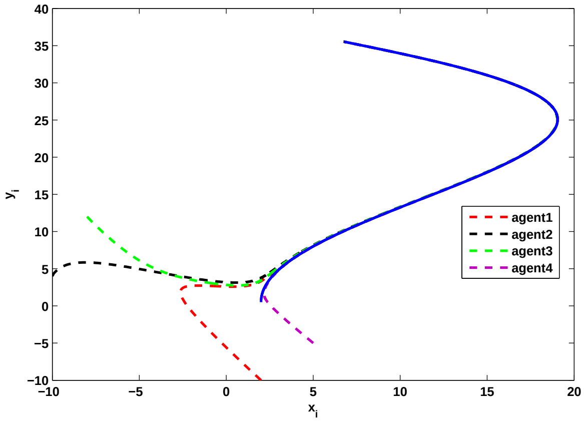


Figure 4.19: Position trajectories of four agents.

At last, for the directed communication graph with six agents, let $\alpha = 0.95$, $\eta = 1.5$, the time-varying reference state and the initial condition are defined as above subsection. As shown in Fig. 4.20 and Fig. 4.21, the system errors approach 0 after a period of time along x-axis and y-axis. And the position trajectories are obtained in Fig. 4.22. It is shown that the consensus is obtained using control law (4.13) and the convergence of fractional-order multi-agent system is faster by comparing to the result with six agents in subsection 4.3.1.

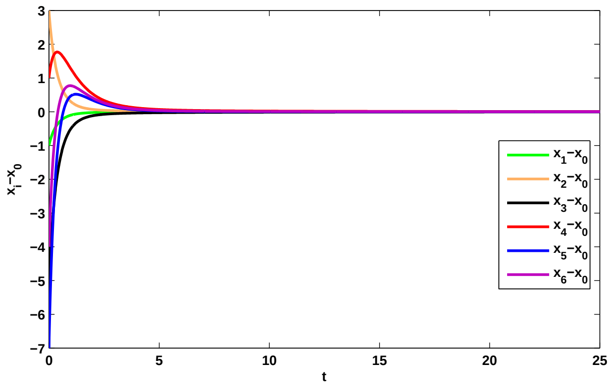


Figure 4.20: The system errors between six agents i ($i = 1, 2, 3, 4, 5, 6$) and the reference state along x-axis using control law (4.13).

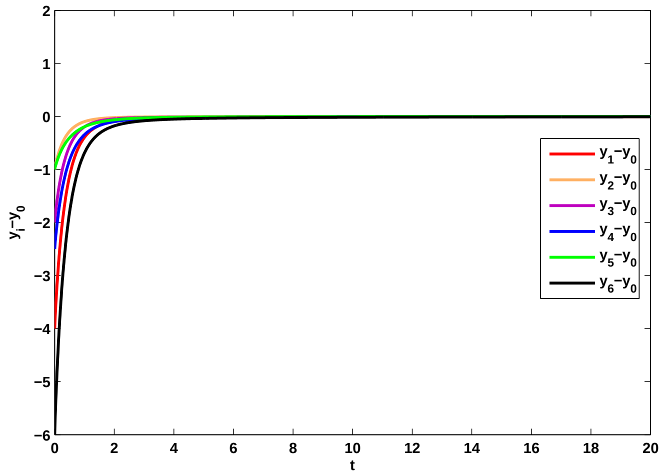


Figure 4.21: The system errors between six agents i ($i = 1, 2, 3, 4, 5, 6$) and the reference state along y-axis using control law (4.13).

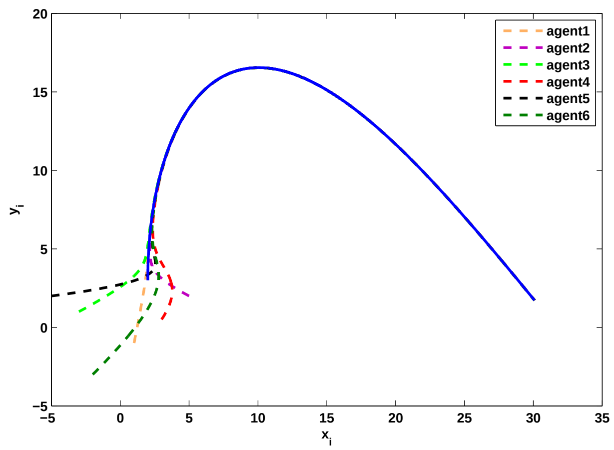


Figure 4.22: Position trajectories of six agents.

4.5 Extension to Formation Tracking

Formation tracking plays an important role in multi-agent coordination. The main idea of formation tracking is to design a control law that enables a group of agents to reach desired formation shapes in the presence of a reference state, which represents the state of common interest to all agents. According to Remark 4.13, the common control law (4.3) can be viewed as a special case of the control law based on error predictor (4.13), therefore, the extended control law (4.13) is studied to achieve the formation tracking problem in this subsection.

Definition 4.14 *For any initial condition $x_i(0)$, $i = 0, 1, \cdots, n$, the formation tracking of fractional-order multi-agent systems can be achieved by control law if the states between agent i and a reference state satisfy the following condition*

$$\lim_{t \to +\infty} (x_i(t) - x_0(t)) = \delta_i, i \in N \tag{4.20}$$

where δ_i denotes a desired state deviation between agent i and a reference state.

Let $\hat{x}_i(t) = x_i(t) - \delta_i$, $\tilde{x}_i(t) = \hat{x}_i(t) - x_0(t)$, and $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (\hat{x}_1(t) - x_0(t), \cdots, \hat{x}_n(t) - x_0(t)^T$. An error predictor is introduced as follows

$$e_i(t) = \eta(\sum_{j=1}^{n} a_{i,j}[\tilde{x}_j(t) - \tilde{x}_i(t)] - a_{i,0}\tilde{x}_i(t)), \tag{4.21}$$

where $\eta > 0$ is a impact factor of the error predictor. Let $e(t) = (e_1(t), \cdots, e_n(t))^T$, then

$$e(t) = \eta(-M\hat{x}(t) + Bx_0(t)\mathbf{1}). \tag{4.22}$$

Based on the above error predictor, the control law based on error predictor is expressed as follows

$$\begin{aligned} u_i(t) = &\sum_{j=1}^{n} a_{i,j}[x_j(t) - x_i(t) + \delta_{ij}] + a_{i,0} \\ &[x_0(t) - x_i(t) + \delta_i] + e_i(t) + u_0(t), i \neq j \end{aligned} \tag{4.23}$$

where $a_{i,j}, i, j \in N$ is the $(i, j)th$ entry of the adjacency matrix A, $a_{i,0}$ is a positive constant if the reference state is available to agent i and $a_{i,0} = 0$ otherwise.

$\delta_{ij} = \delta_i - \delta_j$.

Theorem 4.15 *control law (4.23) solves formation tracking problem, if directed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. *Substituting Eq. (4.23) into the systems (4.1), we can get*

$$\begin{aligned} x^{(\alpha)}(t) =& -M\hat{x}(t) + B(x_0(t)\mathbf{1}) + e(t) + u_0(t) \\ =& -M\hat{x}(t) + B(x_0(t)\mathbf{1}) + e(t) + x_0^{(\alpha)}(t)\mathbf{1}. \end{aligned} \tag{4.24}$$

The system errors can be given as follows

$$\begin{aligned} \tilde{x}^{(\alpha)}(t) =& x^{(\alpha)}(t) - x_0^{(\alpha)}(t)\mathbf{1} \\ =& -M\hat{x}(t) + Bx_0(t)\mathbf{1} + \eta(-M\hat{x}(t) \\ & + Bx_0(t)\mathbf{1}) + x_0^{(\alpha)}(t)\mathbf{1} - x_0^{(\alpha)}(t)\mathbf{1} \\ =& (I + \eta I)(-M\hat{x}(t) + Bx_0(t)\mathbf{1}) \\ =& (I + \eta I)(-(L + B)\hat{x}(t) + Bx_0(t)\mathbf{1}) \\ =& (I + \eta I)(-L(\hat{x}(t) - x_0(t)\mathbf{1}) \\ & - B(\hat{x}(t) - x_0(t)\mathbf{1}) \\ =& -(I + \eta I)M\tilde{x}(t), \end{aligned} \tag{4.25}$$

where $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (\hat{x}_1(t) - x_0(t), \cdots, \hat{x}_n(t) - x_0(t))^T$, $L(\hat{x}(t) - x_0(t)\mathbf{1}) = L\hat{x}(t)$ is applied. Due to directed communication graph $\bar{G}$ has a directed spanning tree, according to the Lemma 4.3, directed communication graph $\bar{G}$ has a globally reachable note v_0, and Lemma 4.8 guarantees that M is positive stable, which means that all the real parts of the eigenvalues of matrix $(I + \eta I)M$ are positive, according to Lemma 4.7, the system errors are asymptotically stable, such that

$$\lim_{t \to +\infty} \tilde{x}_i(t) = 0, i \in N. \tag{4.26}$$

Due to $\tilde{x}_i(t) = \hat{x}_i(t) - x_0$ and $\hat{x}_i(t) = x_i(t) - \delta_i$, that is

$$\lim_{t \to +\infty} (\hat{x}_i(t) - x_0(t)) = 0, i \in N \tag{4.27}$$

and

$$\lim_{t \to +\infty} (x_i(t) - x_0(t)) = \delta_i. \ i \in N \tag{4.28}$$

Then, formation tracking problem of fractional-order multi-agent systems is achieved using control law (4.23) based on error predictor if directed communication graph $\bar{G}$ has a directed spanning tree. ■

To verify the validity of the Theorem 4.15, several simulations are given as follows. Firstly, consider the directed communication graph with three agents shown in Fig. 4.2, which includes a directed spanning tree. Let $\alpha = 0.95$, $\eta = 2$ and the time-varying reference state and the initial condition are chosen as section 4.4. The position trajectories of three agents are shown in Fig. 4.24, where $\delta_1 = (0, 2)^T, \delta_2 = (10\sqrt{3}, -1)^T, \delta_3 = (-10\sqrt{3}, -1)^T$ are chosen. The reference state is denoted by a solid line. From the simulation at $t = \{1s, 10s, 19s, 28s\}$, three agents can follow the time-varying reference state, and form a desired triangle as Fig 4.23 after a period.

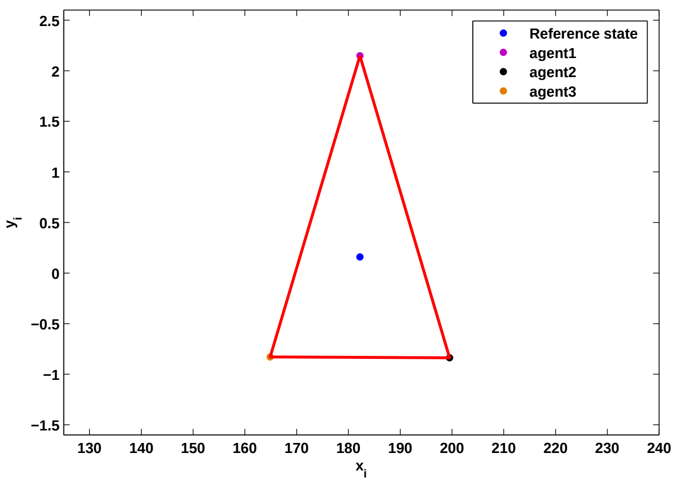


Figure 4.23: The desired formation geometric form for three agents.

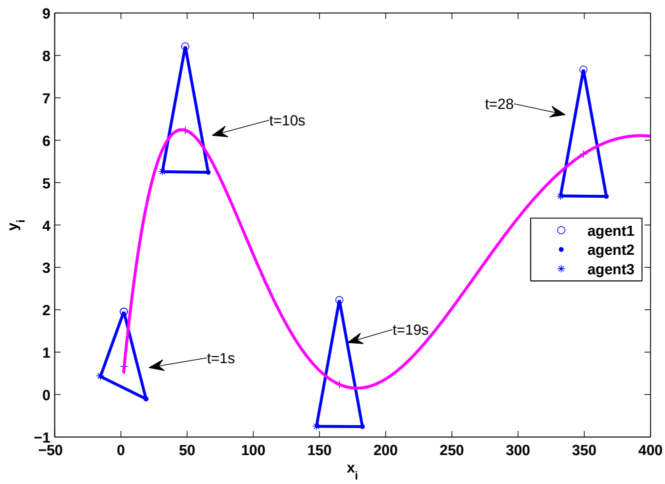


Figure 4.24: Position trajectories of three agents.

Secondly, the directed communication graph with four agents is considered in Fig. 4.6, which comprises a directed spanning tree. Let $\alpha = 0.95$, $\eta = 2.5$, the time-varying reference state and the initial conditions are chosen as section 4.4. The position trajectories of six agents are achieved in Fig. 4.26, where $\delta_1 = (-1, 2)^T, \delta_2 = (1, 0)^T, \delta_3 = (1, -4)^T, \delta_4 = (-1, -2)^T$ are chosen. The reference state is denoted by a solid line. It is shown that the states of all agents converge to the reference state eventually. From the simulation at $t = \{5s, 15s, 25s, 35s, 45s\}$, four agents can follow the time-varying reference state, and form a desired formation geometry as Fig. 4.25 after a period.

Finally, the directed communication graph with six agents is considered in Fig. 4.10, which contains a directed spanning tree. Let $\alpha = 0.95$, $\eta = 1.5$, the time-varying reference state and the initial conditions are chosen as section 4.4. The position trajectories of six agents are achieved in Fig. 4.28, where $\delta_1 = (-0.5, \frac{\sqrt{3}}{2})^T, \delta_2 = (0.5, \frac{\sqrt{3}}{2})^T, \delta_3 = (1, 0)^T, \delta_4 = (-0.5, -\frac{\sqrt{3}}{2})^T, \delta_5 = (-0.5, -\frac{\sqrt{3}}{2})^T, \delta_6 = (-1, 0)^T$ are chosen. The reference state is denoted by a solid line. It is shown that the states of all agents converge to the reference state eventually. From the simulation at $t = \{5s, 15s, 25s, 33s, 45s\}$, six agents can follow the time-varying reference state, and form a regular hexagon as Fig. 4.27 after a period. All the above simulations verify the validity of the Theorem 4.15.

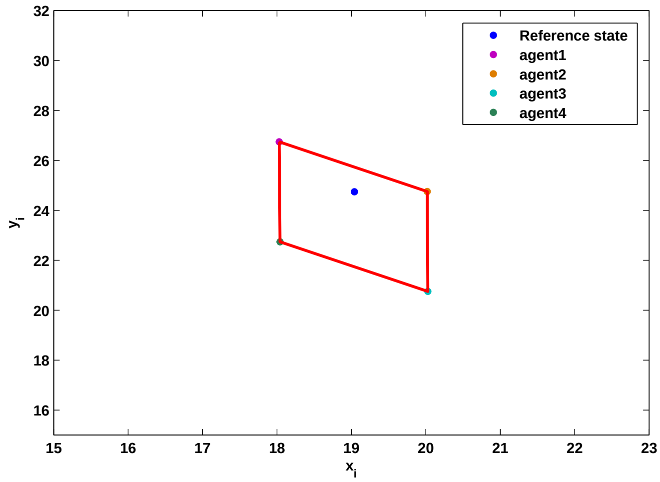


Figure 4.25: The desired formation geometric form for four agents.

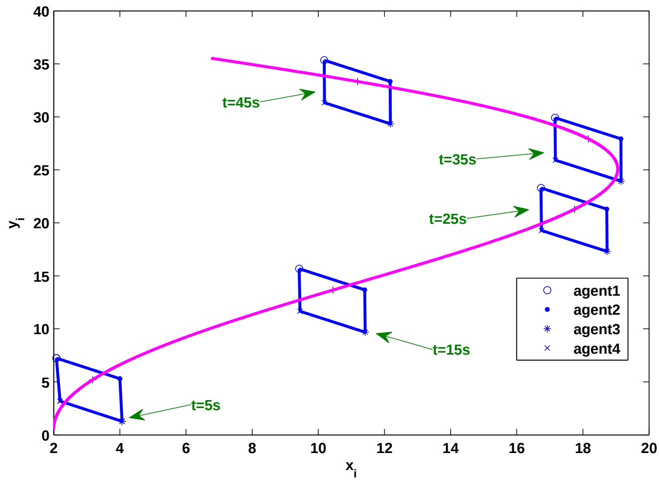


Figure 4.26: Position trajectories of four agents.

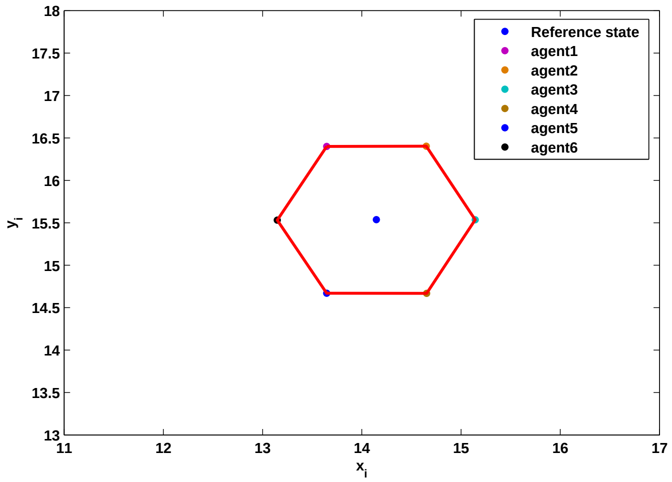


Figure 4.27: The desired formation geometric form for six agents.

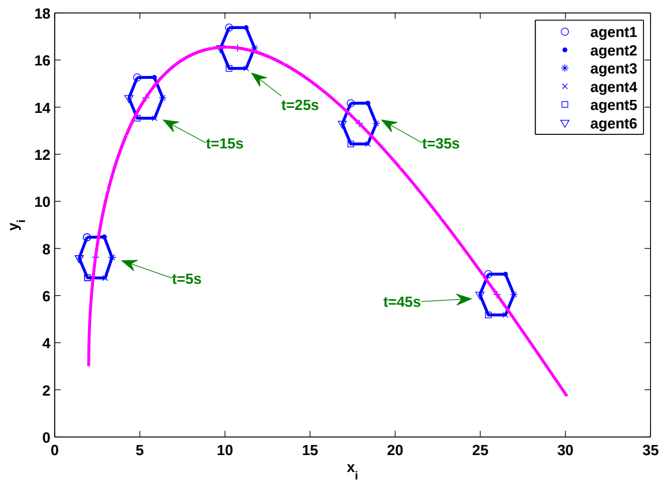


Figure 4.28: Position trajectories of six agents.

4.6 Conclusion

In this chapter, the consensus/formation tracking problems of fractional-order multi-agent systems have been studied. Firstly, the common consensus control law has been given. According to the graph theory and the stability of fractional-order system, it has verified that the control law is effective when a directed communication graph owns a directed spanning tree. Secondly, the consensus control law based on error predictor has been proposed, and it has been shown that the control law based on error predictor is also effective when a directed communication graph has a directed spanning tree. Meanwhile, based on the Mittag-Leffler function, the convergence speed of fractional-order multi-agent systems has been compared using above two types of control laws, it has proved that the convergence of systems is faster using the control law based on the error predictor than by the common one. Thirdly, the control law based on error predictor has been extended to solve the formation tracking problem. At last, several simulations have been presented to verify the effectiveness of the obtained results.

Chapter 5

Consensus/Formation Tracking of Fractional-Order Multi-agent Systems with a Reference State

Contents

5.1 Introduction

We have studied the consensus/formation tracking of fractional-order multi-agent systems with a reference state in Chapter 4 when all agents have access to the reference state. In practice the reference state for the whole team might only be available to only one or some agents. Therefore, in this chapter we shall study consensus/formation with a reference state when only a portion of the agents have access to the reference state. Comparing with existing works, the results of this chapter have the following differences. Firstly, compared with references (Gao *et al.* 2013; Ren 2007), which study the consensus with a reference state base on integer-order multi-agent systems, this chapter considers the consensus with a reference state and formation tracking based on fractional-order multi-agent systems. Secondly, concerning the reference state, we do not require that the reference state information to be available to all followers as in (Hong *et al.* 2006; Xiaohong & Qinghe 2013). As only a portion of the agents in the group can receive the information of time-varying reference state directly, the proposed control laws allow information flow from one agent to others based on directed communication graph to increase redundancy and robustness of the multi-agent systems. This chapter is organized as follows: Firstly, a consensus control law is given for the consensus problem of fractional-order multi-agent systems with a constant reference state. However, this consensus control law cannot guarantee consensus with a time-varying reference state. Thus a general control law and a particular one for consensus with a time-varying reference state of fractional-order multi-agent systems are proposed. We shall prove that if the directed communication graph has a directed spanning tree, all agents can track the time-varying reference state with the proposed control laws. Next, these three control laws are extended to solve the formation tracking problem. Finally, several simulations are presented to verify the validity of the obtained results.

5.2 Preliminaries

In this chapter, the reference state x_0 defined in subsection 1.1.2.1 is represented by vertex v_0, We have a communication graph $\bar{G}$, which consists of above communication graph G, vertex v_0 and edges between a reference state and its neighbors. The reference state is independent and it transmits its state information to its neighbors. The motion of each agent is influenced by the reference state and its neighbors.

5.3 Problem Description

In this chapter, a simple notation $x^{(\alpha)}(t)$ is used to denote the Caputo fractional operator as that in Chapter 1. The dynamics of agent i ($i \in N := \{1, \cdots, n\}$) is described as follows

$$x_i^{(\alpha)}(t) = u_i(t), \quad i \in N \tag{5.1}$$

where $\alpha \in (0, 1]$, $x_i(t) \in R$ and $u_i(t) \in R$ represent the state and the control input of system, $i \in N$, denotes the set of the indexes of agents, $x_i^{(\alpha)}(t)$ is the αth Caputo derivative of $x_i(t)$.

The objective of this chapter is prove that all the agents can track a reference state with local interaction by designing control laws $u_i(t)$. This can also be stated as in Definition 4.4.

5.4 Consensus with a Reference State

In this section, firstly, a consensus control law with a constant reference state is given using graph theory and stability analysis of fractional-order. Next, other control laws are proposed to deal with the consensus problem with time-varying reference state. Finally, the above control laws are extended to solve the formation tracking problem.

5.4.1 Consensus with a constant reference state

In this subsection, the case of consensus with a constant reference state x_0 is considered. The consensus control law is proposed as follows

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}[x_j(t) - x_i(t)] + a_{i,0}[x_0 - x_i(t)], \quad i, j \in N, i \neq j \tag{5.2}$$

where $a_{i,j}, i, j \in N$ is the $(i, j)th$ entry of the adjacency matrix A, $a_{i,0}$ is a positive constant if the reference state is available to agent i and $a_{i,0} = 0$ otherwise.

To study the consensus problem with a reference state, we use the diagonal matrix $B \in R^{n \times n}$ to be a reference state adjacency matrix associated with G as in chapter 4, and $M = L + B$.

Example 5.1 *As shown in Fig. 5.1 and Fig. 5.9, both directed communication graphs have a globally reachable node v_0. Suppose that the weight of each edge is 1 in both cases. Then the Laplace matrix L_1, L_2 and the reference state adjacency matrix B_1, B_2 are easily obtained as follows*

$$L_1 = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, L_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}. \tag{5.3}$$

$$B_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{5.4}$$

Theorem 5.2 *The control law (5.2) solves the consensus problem with a constant reference state if fixed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. Using control law (5.2), the system (5.1) can be written as follows

$$x^{(\alpha)}(t) = -Mx(t) + Bx_0\mathbf{1}, \tag{5.5}$$

where $M = L + B$, $x(t) = (x_1(t), \cdots, x_n(t))^T$, $\mathbf{1} = (1, 1, \cdots, 1)^T$.

To achieve the consensus problem, let $\tilde{x}_i(t) = x_i(t) - x_0$, the system errors can be given as follows

$$\begin{aligned} \tilde{x}^{(\alpha)}(t) =& x^{(\alpha)}(t) - x_0^{(\alpha)}\mathbf{1} \\ =& -Mx(t) + Bx_0\mathbf{1} - x_0^{(\alpha)}\mathbf{1} \\ =& -(L + B)x(t) + Bx_0\mathbf{1} - x_0^{(\alpha)}\mathbf{1} \\ =& -L(x(t) - x_0\mathbf{1}) - B(x(t) - x_0\mathbf{1}) - x_0^{(\alpha)}\mathbf{1} \\ =& -M\tilde{x}(t) - x_0^{(\alpha)}\mathbf{1}, \end{aligned} \tag{5.6}$$

where $\tilde{x}(t) = (\tilde{x}_1(t), \cdots, \tilde{x}_n(t))^T = (x_1(t) - x_0, \cdots, x_n(t) - x_0)^T$, and $L(x(t) - x_0\mathbf{1}) = Lx(t)$ is applied. Because x_0 is a constant, $x_0^{(\alpha)}\mathbf{1} = 0\mathbf{1}$ can be obtained. Hence, the system errors can be rewritten as follows

$$\tilde{x}^{(\alpha)}(t) = -M\tilde{x}(t). \tag{5.7}$$

Since directed communication graph $\bar{G}$ has a directed spanning tree and the reference state is independent, fixed communication graph $\bar{G}$ has a globally reachable node v_0, and Lemma 4.7 guarantees that M is a positive stable matrix, we can get that all the real parts of the eigenvalues of matrix M are positive, which means $|arg(\lambda(-M))| > \alpha\pi/2$. According to Lemma 4.8, the system errors are asymptotically stable, then

$$\lim_{t\to+\infty} \tilde{x}_i(t) = 0. \quad i \in N \tag{5.8}$$

That is

$$\lim_{t\to+\infty} (x_i(t) - x_0(t)) = 0. \quad i \in N \tag{5.9}$$

Hence, consensus with a constant reference state is achieved by control law (5.2) if directed graph $\bar{G}$ has a directed spanning tree . ■

Remark 5.3 *When $\alpha \in (0, 1]$, the system states converge to a constant value related to the initial value in (Cao et al. 2008), but the specification of a particular value is not allowed. By contrast, in this chapter, the system states can converge to any constant point with the consensus control law (5.2) when fixed communication graph $\bar{G}$ contains a directed spanning tree.*

To illustrate the Theorem 5.2, consider a group of three agents with a constant reference state $x_0 = 1$, the initial conditions are chosen as $x_1(0) = -2$, $x_2(0) = 0.5$ and $x_3(0) = 8$. Two cases will be considered in this subsection. Directed ommunication graph Fig. 5.1 includes a directed spanning tree, it is shown in Fig. 5.2 that agents $i, (i = 1, 2, 3)$ converge to the reference constant state after a period of time, which means that the consensus is achieved using the control law (5.2) when the communication graph has a directed spanning tree. On the contrary, in Fig. 5.3, there is no directed spanning tree from node v_0 to other agents v_i, and it is shown from Fig. 5.4 that the agents can't converge to the reference state. The dashed lines represent that states of the tree agents and the solid line represents the constant reference state.

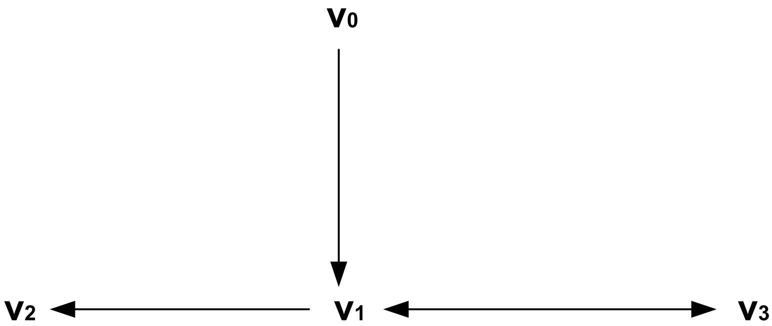


Figure 5.1: Directed communication graph for a group of three agents with a reference state.

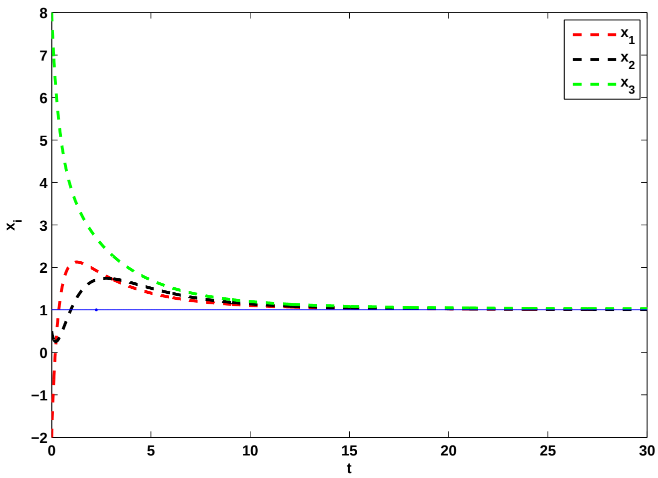


Figure 5.2: The states of agent i with a constant reference state using control law (5.2) under Fig. 5.1.

Fig. 5.5 shows that the system states converge to a constant value 3 using the result in (Cao *et al.* 2008) in Fig. 5.1, the specification of particular value 1 is not allowed. But in this chapter, the agents' states can reach any desired constant state if the fixed communication graph has a directed spanning tree.

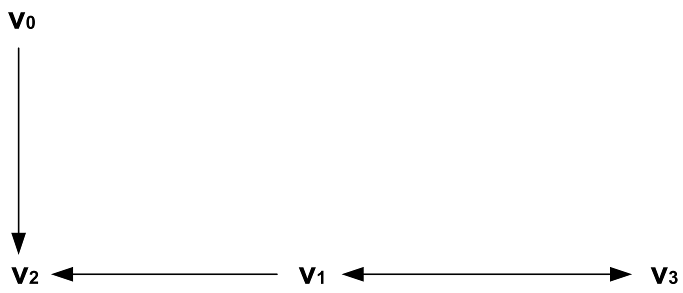


Figure 5.3: Directed communication graph for a group of three agents with a reference state.

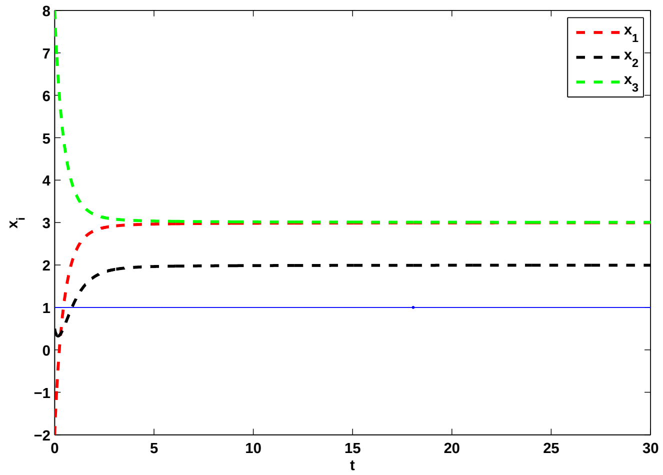


Figure 5.4: The states of agent i with a constant reference state using control law (5.2) in Fig. 5.3.

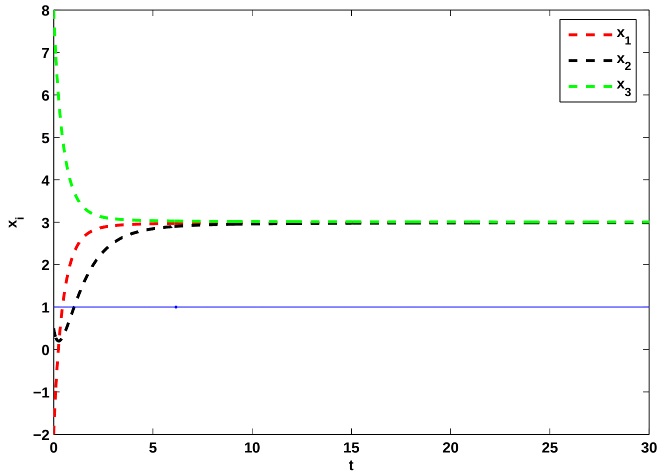


Figure 5.5: The states of agent i without a constant reference state using the result in (Cao *et al.* 2008) in Fig. 5.1.

5.4.2 Consensus with a time-varying reference state

A time-varying reference state is assumed in this subsection. Without loss of generality, the system of time-varying reference state $x_0(t)$ is given as follows

$$x_0^{(\alpha)}(t) = f(t), \tag{5.10}$$

where $f(t)$ is continuous function.

The control law (5.2) is not sufficient for consensus with a time-varying reference state. In order to prove the above point, an example using the consensus control law (5.2) in Fig. 5.1 is given, where $x_0^{(\alpha)}(t) = sin(t)$ is chosen. We can see that the agents i can't converge to the reference state from Fig. 5.6. Therefore, control law (5.2) is invalid to consensus with a time-varying reference state.

When the time-varying information can be received by only a portion of agents according to the information exchange topology, the following consensus control law is given

$$\begin{aligned} u_i(t) = &\frac{1}{\eta_i}\sum_{j=1}^{n} a_{i,j}[x_j^{(\alpha)}(t) - (x_i(t) - x_j(t))] + \\ &\frac{1}{\eta_i} a_{i,0}[x_0^{(\alpha)}(t) - (x_i(t) - x_0(t))],\ \ i,j \in N, i \neq j \end{aligned} \tag{5.11}$$

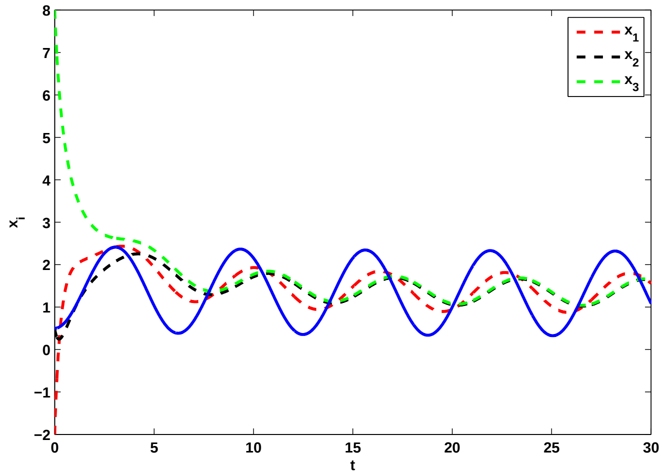


Figure 5.6: The states of agent i with a time-varying reference state using control law (5.2) in Fig. 5.1.

where $a_{i,j}$ and $a_{i,0}$ are defined as in Eq. (5.2), $\eta_i = \sum_{j=1}^{n} a_{i,j} + a_{i,0}$. To control the agents, we just need their local neighbors' information state and their derivatives.

In the particular case when only one agent has access to the time-varying reference state $x_0(t)$, the following consensus control law is also valid

$$\begin{cases} u_i(t) = f(t) - \sum_{j=0}^{n} a_{i,j}(x_i(t) - x_j(t)), \ i \in L \\ u_i(t) = \frac{1}{\sum_{j=0}^{n} a_{i,j}} \sum_{j=0}^{n} a_{i,j}(x_j^{(\alpha)}(t) - (x_i(t) - x_j(t))), \quad i \notin L \end{cases} \tag{5.12}$$

where L denotes the index of the only agent that has access to the reference state $x_0(t)$, and $a_{i,j}$ is defined as in Eq. (5.2).

Theorem 5.4 *The control laws (5.11) and (5.12) solve the consensus problem with a time-varying reference state if fixed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. According to the consensus control law (5.11), the dynamics can be written as follows

$$
\begin{aligned}
x_i^{(\alpha)}(t) &= \frac{1}{\eta_i}\sum_{j=1}^{n} a_{i,j}[x_j^{(\alpha)}(t) - (x_i(t) - x_j(t))] + \frac{1}{\eta_i} a_{i,0}[x_0^{(\alpha)}(t) - (x_i(t) - x_0(t))] \\
&= \frac{1}{\sum_{j=0}^{n} a_{i,j}} \sum_{j=0}^{n} a_{i,j}[x_j^{(\alpha)}(t) - (x_i(t) - x_j(t))]. \quad i, j \in N, i \neq j \qquad (5.13)
\end{aligned}
$$

After some manipulations, the following relation can be given

$$
\sum_{j=0}^{n} a_{i,j}[x_i^{(\alpha)}(t) - x_j^{(\alpha)}(t)] = -\sum_{j=0}^{n} a_{i,j}[x_i(t) - x_j(t)]. \quad i, j \in N, i \neq j \qquad (5.14)
$$

According to Lemma 4.8, the following result can be obtained

$$
\sum_{j=0}^{n} a_{i,j}[x_i(t) - x_j(t)] \to 0, \quad i, j \in N, i \neq j. \qquad (5.15)
$$

Note that there are n equations with $n+1$ variables in Eq. (5.15), when the reference state is added to communication digraph, the equation $0 = 0$ also can be added. Eq. (5.15) can be rewritten in matrix form as $L_{n+1}x^{'} \to 0$, where $x^{'} = [x_1, x_2, \cdots, x_n, x_0]^T$, which is associated to directed communication graph $\bar{G}$. According to Lemma 1.6, $x_i \to x_0$, $i \in N$ can be obtained.

For the consensus control law (5.12), the second equation in Eq. (5.12) can be written as

$$
x_i^{(\alpha)}(t) = \frac{1}{\sum_{j=0}^{n} a_{i,j}} \sum_{j=0}^{n} a_{i,j}(x_j^{(\alpha)}(t) - (x_i(t) - x_j(t))). \quad i \notin L \qquad (5.16)
$$

After some computation, the following form equation can be obtained

$$
\sum_{j=0}^{n} a_{i,j}(x_i^{(\alpha)}(t) - x_j^{(\alpha)}(t)) = -\sum_{j=0}^{n} a_{i,j}(x_i(t) - x_j(t)). \quad i \notin L \qquad (5.17)
$$

The eigenvalues of system matrix A in (5.17) are -1 , which means $|arg(\lambda(A))| > \alpha\pi/2$. According to Lemma 4.8, $\sum_{j=0}^{n} a_{i,j}(x_i(t) - x_j(t)) \to 0$ can be obtained. $x_i(t) \to x_j(t), i, j \in N$, if directed communication graph $\bar{G}$ includes a directed spanning tree. According to the first equation in (5.12) and Lemma 4.8, $x_i(t) \to x_0(t), \forall i \in N$. ■

Remark 5.5 *The control laws (5.11) and (5.12) allow some agents to access the time-varying reference state directly, and the information of the reference state can flow from one agent to other agents according to the directed communication graph.*

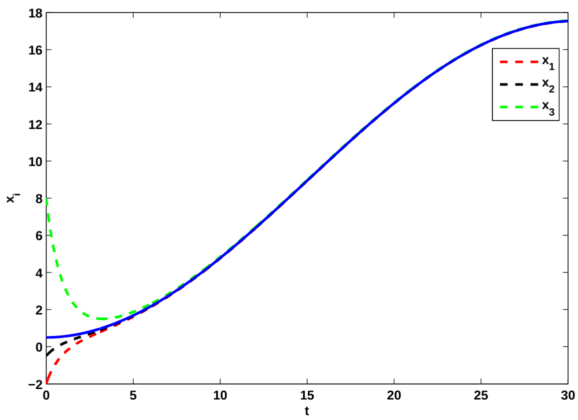


Figure 5.7: The states of three agents with a time-varying reference state using control law (5.11).

To illustrate the effectiveness of the consensus control law (5.11) in Theorem 5.4, directed communication graph Fig. 5.1 is used. Let $\alpha = 0.95$ and the time-varying reference state $x_0^{(\alpha)}(t) = sin(t/10)$. From Fig. 5.7, it is shown that all the states of agents converge to the reference state when the fixed communication graph includes a directed spanning tree. The reference state is denoted by a solid line, and all the states of agents are denoted by dashed lines. The errors between agents i and the reference state are shown in Fig. 5.8, which satisfies the Definition 4.4 of the consensus problem.

When more than one agent have access to the reference state, directed communication graph Fig. 5.9 is applied, where agent 1 and agent 4 have access to the reference state directly. Let $\alpha = 0.9$ and $x_0^{(\alpha)}(t) = cos(\pi t/10)$. From Fig. 5.10, the agents can follow the reference state eventually, where the reference state is denoted by a solid line, and the states of all agents are denoted by dashed lines. The errors between agents i and the reference state are shown in Fig. 5.11, the Definition 4.4 of the consensus problem is satisfied. It is shown that the consensus can be achieved using the control law (5.11) when the directed communication graph has a directed spanning tree.

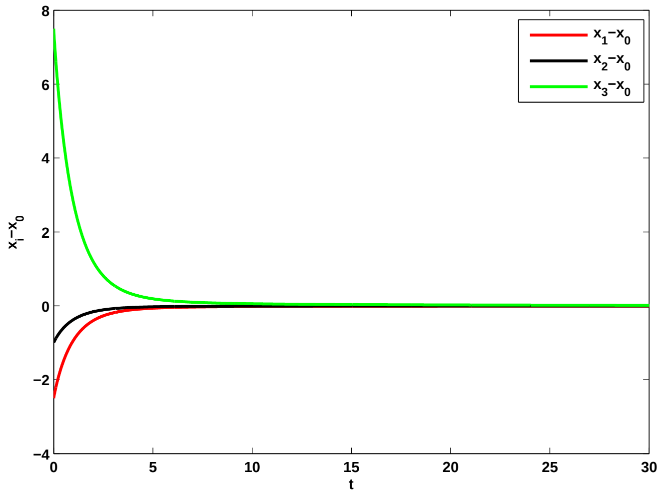


Figure 5.8: The errors between three agents i and the reference state using control law (5.11).

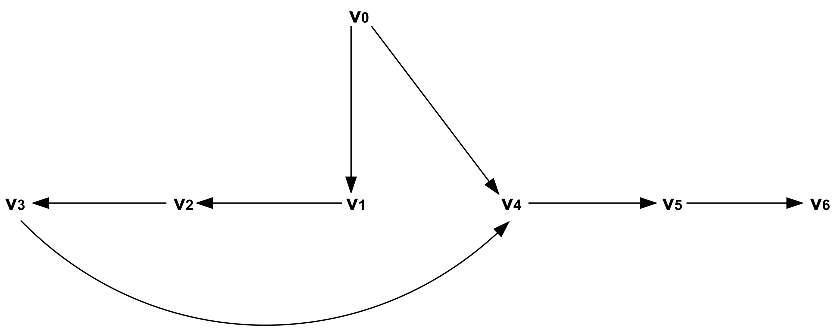


Figure 5.9: Directed communication graph for a group of six agents with a reference state.

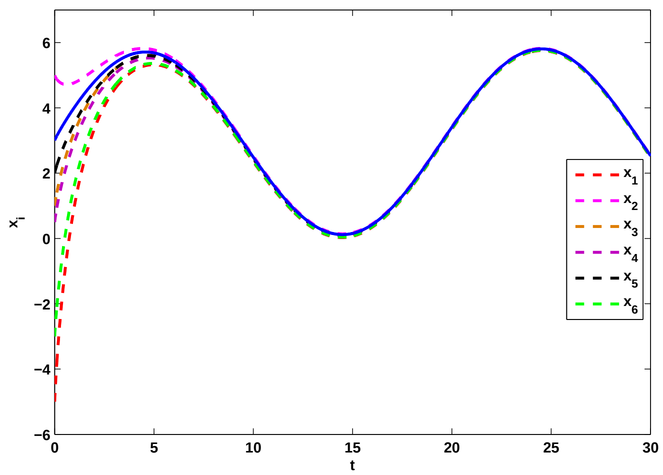


Figure 5.10: The states of six agents with a time-varying reference state using control law (5.11).

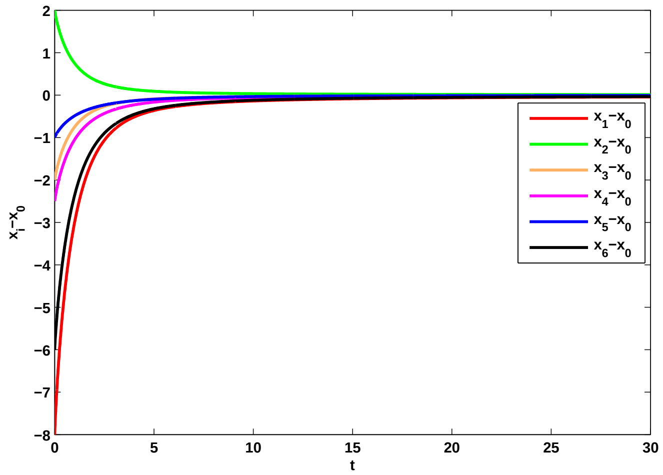


Figure 5.11: The errors between six agents i and the reference state using control law (5.11).

For only one agent has access to the reference state, the control law (5.12) is used to solve the problem. Fixed communication graph Fig. 5.1 is considered, and $x_0^{(\alpha)}(t) = sin(t/10)$ is chosen. All the states of agents converge to the reference state eventually in Fig. 5.12, and the reference state is denoted by a solid line, and all the states of agents are denoted by dashed lines. The errors between agents i and the reference state are also shown in Fig. 5.13, and the condition $\lim_{t\to+\infty}(x_i(t) - x_0(t)) = 0$ in the Definition 4.4 is verified, which means that the control law (5.12) is effective.

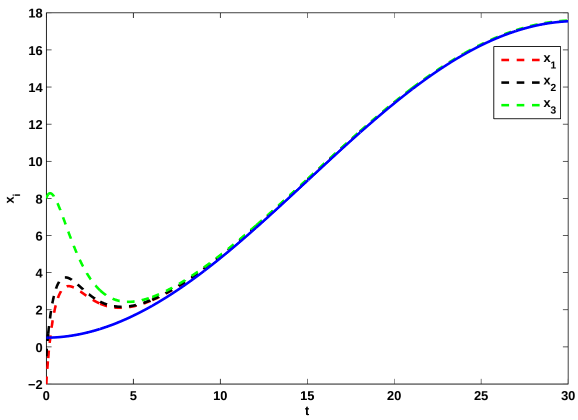


Figure 5.12: The states of three agents with a time-varying reference state using control law (5.12).

5.4.3 Extensions to formation tracking

In practice, the desired formation geometries between agents are often needed (Ni & Cheng 2010; Peng *et al.* 2013b). The consensus with a reference state is considered as a part of the formation tracking problem, the latter demands both tracking and relative position keeping. Therefore, in this subsection, the extended control laws of Eqs. (5.2), (5.11) and (5.12) are considered to guarantee that the formation tracking problem can be achieved. The definition of the formation tracking is given as Definition 4.14

Firstly, the extended consensus control law of Eq. (5.2) in subsection 5.2.1 is

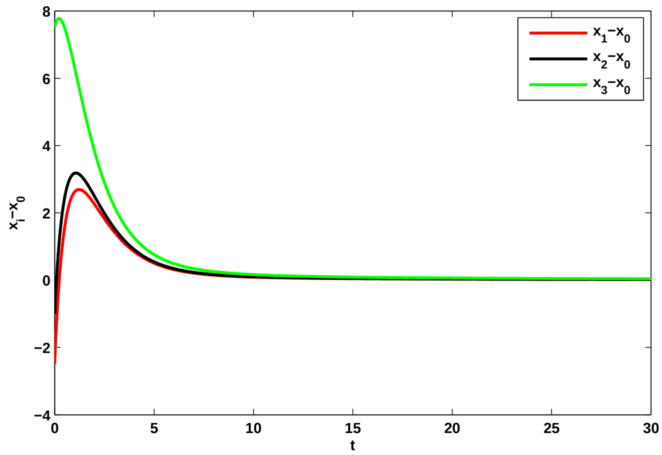


Figure 5.13: The errors between three agents i and the reference state using control law (5.12).

given as follows

$$u_i(t) = \sum_{j=1}^{n} a_{i,j}(x_j(t) - x_i(t) + \delta_{ij}) + a_{i,0}(x_0 - x_i(t) + \delta_i), \quad i, j \in N, i \neq j \quad (5.18)$$

where $\delta_{ij} = \delta_i - \delta_j$ and x_0 are constant values.

Theorem 5.6 *The control law (5.18) solves the formation tracking problem with a constant reference state if directed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. With the consensus control law (5.18), Eq. (5.1) can be written as the following form

$$\hat{x}^{(\alpha)}(t) = -M\hat{x}(t) + Bx_0\mathbf{1}, \quad (5.19)$$

where $\hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_n(t)]^T$, $\hat{x}_i(t) = x_i(t) - \delta_i$. Then, the system errors can be written as the same form as Eq. (5.6). According to Lemma 4.8,

$$\lim_{t\to+\infty} (\hat{x}_i(t) - x_0(t)) = 0. \quad i \in N \quad (5.20)$$

That is

$$\lim_{t\to+\infty} (x_i(t) - x_0(t)) = \delta_i. \quad i \in N \quad (5.21)$$

According to the definition of the formation tracking, the proof of the theorem is completed. ■

For control laws (5.11) and (5.12) with time-varying reference state $x_0(t)$ in subsection 5.2.2, the extended control laws can be given respectively as follows

$$\begin{aligned}u_i(t)=&\frac{1}{\eta_i}\sum_{j=1}^{n}a_{i,j}[x_j^{(\alpha)}(t)-(x_i(t)-x_j(t)-\delta_{ij})]+\\&\frac{1}{\eta_i}a_{i,0}[x_0^{(\alpha)}(t)-(x_i(t)-x_0(t)-\delta_i)],\quad i,j\in N, i\neq j\end{aligned}\tag{5.22}$$

$$\begin{cases}u_i(t)=f(t)-\sum_{j=0}^{n}a_{i,j}(x_i(t)-x_j(t)-\delta_{ij}),\ i\in L\\ u_i(t)=\frac{1}{\sum_{j=0}^{n}a_{i,j}}\sum_{j=0}^{n}a_{i,j}(x_j^{(\alpha)}(t)-(x_i(t)-x_j(t)-\delta_{ij})),\quad i\notin L\end{cases}\tag{5.23}$$

where $a_{i,j}$, $a_{i,0}$ and η_i have the same meaning as in Eq. (5.11). The following Theorem is valid.

Theorem 5.7 *The control laws (5.22) and (5.23) solve the formation tracking problem with a time-varying reference state if directed communication graph $\bar{G}$ has a directed spanning tree.*

Proof. Define $\hat{x}_i(t)=x_i(t)-\delta_i$. Using the extended control law (5.22), Eq. (5.1) can be given as follows

$$\hat{x}_i^{(\alpha)}(t)=\frac{1}{\eta_i}\sum_{j=1}^{n}a_{i,j}[\hat{x}_j^{(\alpha)}(t)-(\hat{x}_i(t)-\hat{x}_j(t))]+\frac{1}{\eta_i}a_{i,0}[x_0^{(\alpha)}(t)-(\hat{x}_i(t)-x_0(t))].\tag{5.24}$$

After some manipulations, the same relationship as Eq. (5.14) can be got. Then, using the same method as in Theorem 5.4,

$$\lim_{t\to+\infty}(\hat{x}_i(t)-x_0(t))=0,\quad i\in N\tag{5.25}$$

can be obtained, which implies

$$\lim_{t\to+\infty}(x_i(t)-x_0(t))=\delta_i.\quad i\in N\tag{5.26}$$

The formation tracking is achieved. For the extended consensus control law (5.23), the same process can be applied to verify its effectiveness when only one agent can access to the reference state. ■

Remark 5.8 $\alpha = 1$ *can be chosen, the existing consensus with a reference state for integer-order multi-agent systems (Ren 2007) can be viewed as a special case of this chapter.*

Remark 5.9 *When* $\delta_i = 0$*, the formation tracking problem becomes the consensus problem, which means that the consensus with reference state can be viewed as a part of the formation tracking problem in this chapter.*

To verify the effectiveness of the extended consensus control law, several simulations are given in two dimensional space. Firstly, consider control law (5.18) and directed communication graph Fig. 5.1. Let $\alpha = 0.95$, $\delta_1 = [0, 1]^T$, $\delta_2 = [\sqrt{3}/2, -0.5]^T$ and $\delta_3 = [\sqrt{3}/2, -0.5]^T$. Choose the reference state be $x_0 = (2, 4)^T$. The desired geometry is chosen by a triangle as shown in Fig. 5.14. It is shown from the simulation at $t = \{0s, 5s, 10s, 15s, 20s, 25s, 30s\}$ in Fig. 5.15 that the three agents can follow the constant reference state, and converge to the desired formation geometry. The effectiveness of Theorem 5.6 is verified.

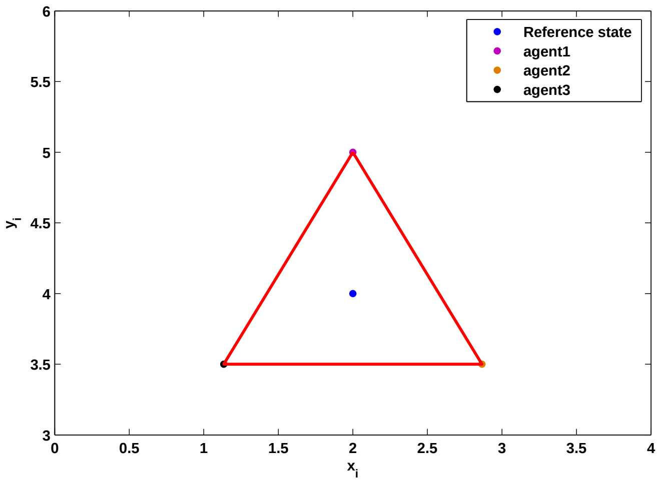


Figure 5.14: The desired formation geometric form for three agents using control law (5.18).

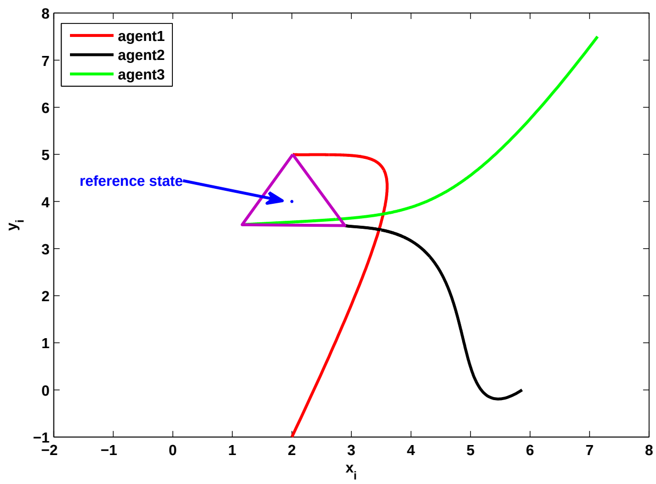


Figure 5.15: Formation geometrics with a constant state using control law (5.18).

Secondly, when the reference state is time varying, control law (5.22) in directed communication graph Fig. 5.1 are used. Let $\alpha = 0.95$, $\delta_1 = [0, 1]^T$, $\delta_2 = [\sqrt{3}/2, -0.5]^T$, and $\delta_3 = [\sqrt{3}/2, -0.5]^T$. The time-varying reference state is chosen as $x_0^{(\alpha)}(t) = [2cos(t/50), sin(t/20)]^T$. From the simulation at $t = \{0s, 5s, 10s, 15s, 20s, 25s, 30s\}$ in Fig. 5.17 that the three agents can follow the time-varying reference state, and converge to the desired formation geometry as in Fig. 5.16 after a period.

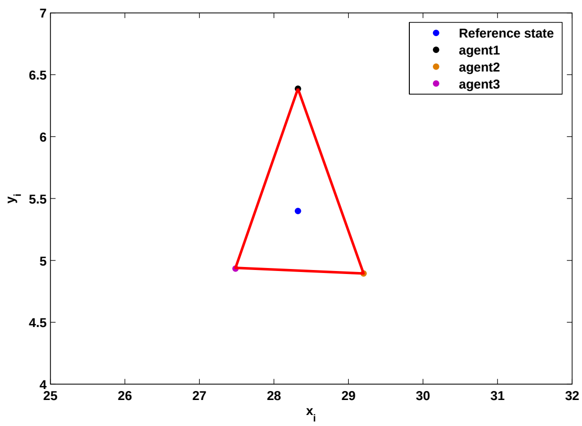


Figure 5.16: The desired formation geometric form for three agents using control law (5.22).

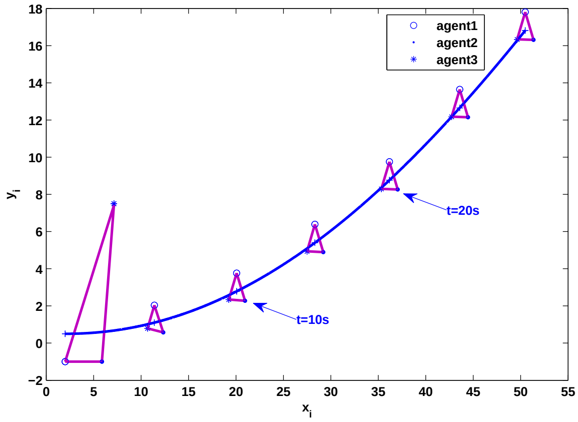


Figure 5.17: Formation geometrics of three agents with a time-varying state using control law (5.22).

Furthermore, when more than one agent have access to the information of time-varying reference state, directed communication graph Fig. 5.9 with six agents is considered using control law (5.22). Let $\alpha = 0.9$, $\delta_1 = [-0.5, \sqrt{3}/2]^T$, $\delta_2 = [0.5, \sqrt{3}/2]^T$, $\delta_3 = [1, 0]^T$, $\delta_4 = [0.5, -\sqrt{3}/2]^T$, $\delta_5 = [-0.5, -\sqrt{3}/2]^T$, and $\delta_6 = [-1, 0]^T$. The time-varying reference state is chosen as $x_0^{(\alpha)}(t) = [2cos(t/20), cos(\pi t/10)]^T$. The desired geometry is a regular hexagon, from the simulation at $t = \{0s, 5s, 10s, 15s, 20s, 25s, 30s\}$ in Fig. 5.19, six agents can follow the time-varying reference state, and converge to the regular hexagon as in Fig. 5.18 after a period of time.

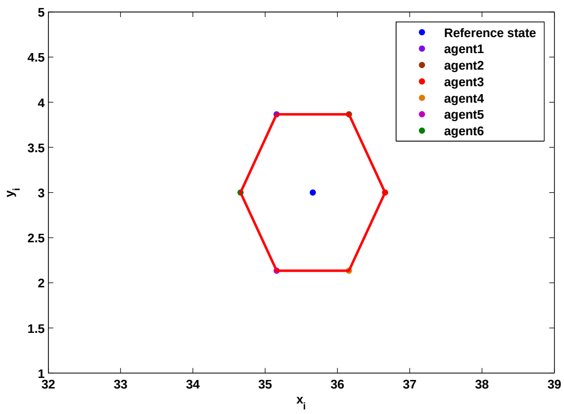


Figure 5.18: The desired formation geometric form for six agents using control law (5.22).

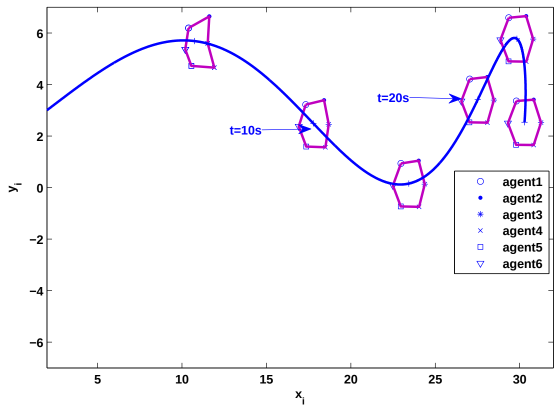


Figure 5.19: Formation geometrics of six agents with a time-varying state using control law (5.22).

Finally, for the particular case, directed communication graph Fig. 5.1 is used to verify the effectiveness of control law (5.23). Let $\alpha = 0.95$, $\delta_1 = [0, 1]^T$, $\delta_2 = [\sqrt{3}/2, -0.5]^T$, and $\delta_3 = [\sqrt{3}/2, -0.5]^T$. Choose the time-varying reference state as $x_0^{(\alpha)}(t) = [20sin(t/100), sin(t/10)]^T$. From the simulation at $t = \{5s, 10s, 15s, 20s, 25s, 30s\}$ in Fig. 5.21, three agents can follow the time-varying reference state, and the desired formation geometry of tree agents as in Fig. 5.20 is formed. That is, the effectiveness of the Theorem 5.7 is confirmed.

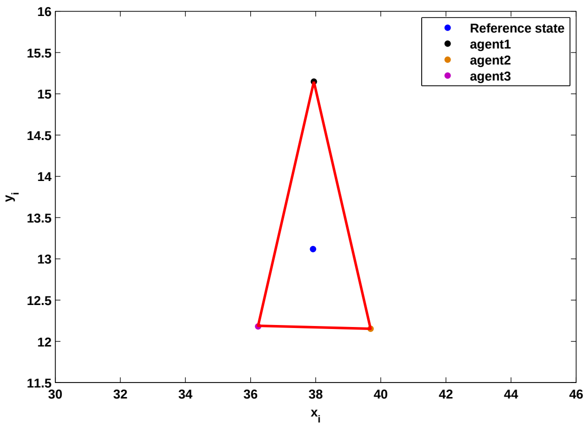


Figure 5.20: The desired formation geometric form for three agents using control law (5.23).

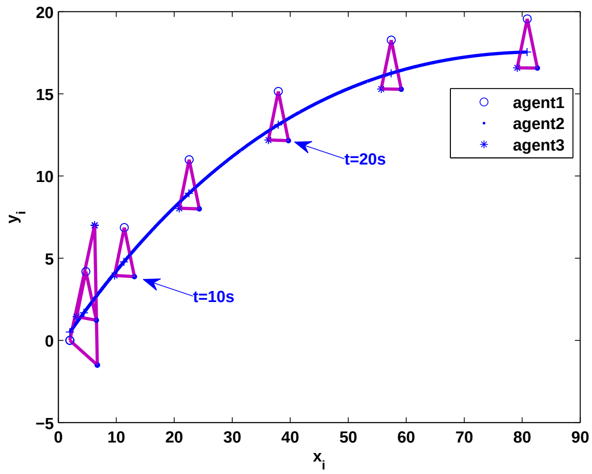


Figure 5.21: Formation geometrics of three agents with a time-varying state using control law (5.23).

5.5 Conclusion

In this chapter, consensus problem of fractional-order multi-agent systems with a reference state has been studied under directed communication graph. Firstly, consensus of fractional-order multi-agent systems with a constant reference state has been studied. A control law has been provided for consensus tracking using graph theory and stability theory of fractional-order system. Secondly, the control laws have been proposed to solve the consensus of fractional-order multi-agent systems with a time-varying reference state. Using graph theory and stability analysis of fractional-order system, a theorem has been given to judge the effectiveness of the control laws. Finally, these control laws have been extended to address the formation tracking problem. The relative theorems have been proved guaranteeing the achievement of the formation tracking. The simulations have been provided to verifying the validity of the above results. Since it is not practical to use the above control laws which contain state derivatives, more simple control laws will be a subject for our future work.

Conclusions and Direction for Future Work

The purpose of this chapter is to summarize the contributions of this research work and to introduce our future works for completing and improving the obtained results.

Summary of main results

In this thesis, we have studied coordination of fractional-order multi-agent systems. We mainly focused on formation producing problem and consensus/formation tracking problem.

After some definitions and applications have been introduced in chapter 1, the formation producing of fractional-order multi-agent systems with absolute damping and communication-delay has been investigated in chapter 2. Firstly, the fractional-order multi-agent systems and the control law have been established, and using the vector conversion, the nonlinear systems have been changed into linear systems. Then, using the matrix theory, graph theory and the frequency domain analysis, we have established a theorem showing that formation control would be achieved if the following conditions are guaranteed: $\alpha \in (0, 2)$, the weighted communication topology has a directed spanning tree and all the roots of characteristic equation have negative real parts or equate to zero. Finally, the simulation case with two agents with directed communication graph has been performed to show that the integer-order systems are the special cases of fractional-order systems. Furthermore, formation control of three agents and six agents systems have been achieved to validate the effectiveness of our theoretical analysis. Comparing with published existing works, this research work has the following advantages: Firstly, instead of distributed multi-agent coordination systems with linear dynamics, we have studied nonlinear multi-agent system with fractional-order absolute damping. Secondly, although time delay is a very important aspect in practical applications, there are few results on formation control of fractional-order multi-agent systems with time delay, we have included the time delay case in our study for the formation producing of fractional-order multi-agent systems. Finally, whereas existing results on the stability analysis of equilibrium points is studied using Lyapunov method, the frequency-domain analysis method has been used for stability analysis.

In Chapter 3, a control law with relative damping and communication-delay has been designed for the formation producing problem. Different from the formation producing with absolute damping in 2, all agents achieve formation asymptotically with zero final velocities, we realized formation producing with all agents moving as a group, instead of rendezvous at a stationary point has been achieved. In this case, only relative measurements (position or velocity) are needed, knowing that it is more difficult to achieve formation producing with relative damping. Firstly, a distributed formation control law with communication delay has been given under directed interaction graph. Secondly, stability conditions for formation producing of fractional-order multi-agent systems with relative damping and communication delay have been established using the frequency-domain analysis method. Finally, to illustrate the obtained results, several simulations have been presented based on predictor-corrector method. the same advantages as the results in chapter 2. Meanwhile, different from the above results, agents can converge to stationary point, in this chapter, agents can move as a group in the presence of communication delay.

In chapter 4, the consensus tracking of fractional-order multi-agent systems has been considered. Noting that in chapters 2 and 3 we have studied formation producing without a reference state, its final target value to be reached is inherent. However, it is desirable that the states of all agents can asymptotically track a reference state, which represents the state of common interest to all agents. Therefore, in chapter 4 we have investigated consensus tracking with a reference state. Firstly, a common control law has been proposed, and validated when the communication graph has a directed spanning tree. Secondly, a control law based on error predictor has been proposed and validated when the communication graph has a directed spanning tree. Then the convergence speeds of fractional-order multi-agent systems based on the above control laws have been compared. It is verified that the convergence of systems is faster using the control law based on error predictor than the one using the first one. Thirdly, the control law based on error predictor has been extended to solve the formation tracking problem. Finally, several simulations have been presented to verify the effectiveness of the obtained results. Comparing with existing results, this chapter has the following differences. Firstly, in contrast to the studies without a reference state, this chapter considered the consensus of multi-agent systems with a reference state. Secondly, different from the results on coordination of integer-order multi-agent systems, the consensus tracking of multi-agent systems has been studied based on fractional-order systems. Two control laws have been proposed. Finally, the obtained convergence speeds corresponding to the proposed two control laws are compared.

In chapter 5, we continued studying the consensus tracking of fractional-order multi-agent systems in the presence of a reference state. Compared with the results in chapter 4, the reference state for the whole group is only available to one or some agents. Firstly, a consensus control law has been given to address the consensus tracking problem with a constant reference state. However, it has been shown that the consensus control law cannot guarantee consensus with a time-varying reference state. Therefore, a general control law and a particular one for consensus with a time-varying reference state of fractional-order multi-agent systems have been proposed. It has been established that if the directed communication graph has a directed spanning tree, all agents can track the time-varying reference state with the proposed control laws. Next, the above control laws have been extended to address the formation tracking problem. Finally, several simulations have been conducted to verify the effectiveness of the obtained results. Comparing with existing works, the results obtained have the following differences. Firstly, the consensus with a reference state is applicable to integer-order multi-agent systems, whereas we have studied the consensus with a reference state and formation tracking based on fractional-order multi-agent systems. Secondly, we do not require that the information of reference state is available to all followers. But only a portion of the agents in the group can receive the information of time-varying reference state directly.

Future works

The following study directions and areas are under consideration.

In chapters 2 and 3, the methods to evaluate the formation producing is complex when a large number of agents are considered, hence, more simple methods to evaluate the problem will be part of our future works. In chapter 5, the control laws to solve the consensus tracking problem contain state derivatives, which are difficult to be applied in practice, therefore, more simple control laws will be a topic for future research.

Note that although we focus on fixed directed communication graph in this thesis, the analysis of proposed control laws can be extended to switching directed communication graph. Furthermore, the issues of disturbance, effects of multi-agent systems, and multi reference states also need to be addressed.

Our present works mainly focus on theoretical aspects, and only numerical simulations have been conducted to verify the effectiveness of the results. In the future, we plan to testing our results using a platform of real robots.

Appendix A

The Numerical Method of Predictor-Corrector

To simulate the fractional-order system, the Predictor-Corrector method is used (Bhalekar 2013). Firstly, Consider the following fractional-order initial problem

$$\begin{cases} x^{(\alpha)}(t) = f(t, x(t)),\ t \in [0, T], \quad 0 < \alpha \leq 1 \\ x^{(k)}(0) = x_0(k), k = 0, 1, , m-1, \alpha \in (m-1, m] \end{cases} \tag{A.1}$$

where $x(t)$ is a time dependent function, T is the simulation bound of time. The initial problem (A.1) is equivalent to the Volterra equation,

$$x(t) = x(0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, x(\tau)) d\tau. \tag{A.2}$$

Consider an uniform grid $\{t_n = nh : n = 0, 1, \cdots, N\}$ for some N and $h = T/N$. Let $x_h(t_n)$ denotes the approximation of $x(t_n)$. Assume that we have already calculated approximation $x_h(t_j), j = 1, 2, \cdots, n$ and want to obtain $x_h(t_{n+1})$ by the following equation

$$\begin{aligned} x_h(t_{n+1}) =& x(0) + \frac{h^\alpha}{\Gamma(\alpha+2)} f(t_{n+1}, x_h^P(t_{n+1})) \\ &+ \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f(t_j, x_h(t_j)), \end{aligned} \tag{A.3}$$

where $a_{j,n+1}$ are given by

$$a_{j,n+1} = \begin{cases} n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha}, & if\ j=0 \\ (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}, & if\ 1 \leq j \leq n \\ 1. & if\ j=n+1 \end{cases} \tag{A.4}$$

The preliminary approximation $x_h^P(t_{n+1})$ is called predictor and is given by

$$x_h^P(t_{n+1}) = x(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, x_h(t_j)), \tag{A.5}$$

where $b_{j,n+1}$ are given as follows

$$b_{j,n+1} = \frac{h^{\alpha}}{\alpha}((n+1-j)^{\alpha} - (n-j)^{\alpha}). \tag{A.6}$$

Error in this method is

$$max_{j=0,1,\cdots,N}|x(t_j) - x_h(t_j)| = O(h^p), \tag{A.7}$$

where $p = min(2, 1+\alpha)$.

A.0.0.1 The numerical method of predictor-corrector with communication delay

In order to simulate the fractional-order multi-agent systems with communication delay, we consider the following fractional-order differential equation with communication delay

$$\begin{cases} {}_a^C D_t^{\alpha} x(t) = f(t, x(t), x(t-\tau)), & t \in [0, T], 0 < \alpha \leq 1 \\ x(t) = g(t), & t \in (-\tau, 0] \end{cases} \tag{A.8}$$

where $x(t)$ represents the state of systems and has continuous derivative, $f(t, x(t), x(t-\tau))$, $g(t)$ are real functions in this paper, and τ represents the communication delay, T is the simulation bound of time.

Consider a uniform grid $\{t_n = nh : n = -k, -k+1, \cdots, -1, 0, 1, \cdots, N\}$ where k and N are integers such that $h = T/N$ and $h = \tau/k$. Let

$$x_h(t_j) = g(t_j), j = -k, -k+1, \cdots, -1, 0 \tag{A.9}$$

and note that

$$x_h(t_j-\tau)=x_h(jh-kh)=x_h(t_{j-k}).\ \ j=0,1,\cdots,N \tag{A.10}$$

Suppose we have already calculated approximations $x_h(t_j)\approx x(t_j)$, $(j=-k,-k+1,\cdots,-1,0,1,\cdots,n)$ and we want to calculate $x_h(t_{n+1})$ using

$$x(t_{n+1})=g(0)+\frac{1}{\Gamma(\alpha)}\int_0^{t_{n+1}}(t_{n+1}-\xi)^{\alpha-1}f(\xi,x(\xi),x(\xi-\tau)d\xi. \tag{A.11}$$

Note that Eq. (A.11) is obtained by applying $I^{\alpha}_{t_{n+1}}$ on both sides of fractional-order differential Eq. (A.8):

$$I^{\alpha}_{t_{n+1}}\,{}^{C}_{a}D^{\alpha}_{t_{n+1}}x(t_{n+1})=x(t_{n+1})-\sum_{k=0}^{m-1}\frac{d^kx(0)}{dt^k_{n+1}}\frac{t^k_{n+1}}{k!}, \tag{A.12}$$

$$I^{\alpha}_{t_{n+1}}f(t_{n+1})=\frac{1}{\Gamma(\alpha)}\int_0^{t}(t_{n+1}-\tau)^{\alpha-1}f(\tau)d\tau, t>0 \tag{A.13}$$

where $m-1<\alpha\leq m$.

We use approximations $x_h(t_n)$ for $x(t_n)$ in Eq. (A.11). Further the integral in Eq. (A.11) is evaluated using product trapezoidal quadrature formula. The corrector formula is thus

$$\begin{aligned}x_h(t_{n+1})=&g(0)+\frac{h^\alpha}{\Gamma(\alpha+2)}f(t_{n+1},x_h(t_{n+1}),x_h(t_{n+1}-\tau))\\&+\frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n}a_{j,n+1}f(t_j,x_h(t_j),x_h(t_j-\tau))\\=&g(0)+\frac{h^\alpha}{\Gamma(\alpha+2)}f(t_{n+1},x_h(t_{n+1}),x_h(t_{n+1-k}))\\&+\frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n}a_{j,n+1}f(t_j,x_h(t_j),x_h(t_{j-k})),\end{aligned} \tag{A.14}$$

where $a_{j,n+1}$ are given by

$$a_{j,n+1} = \begin{cases} n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha}, & if j = 0 \\ (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}, & if 1 \leq j \leq n \\ 1. & if j = n+1 \end{cases} \tag{A.15}$$

The unknown term $x_h(t_{n+1})$ appears on both sides of Eq. (A.14) and due to nonlinearity of $f(t)$ in Eq. (A.14) can not be solved explicitly for $x_h(t_{n+1})$. So we replace the term $x_h(t_{n+1})$ on the right hand side by an approximation $x_h^P(t_{n+1})$, called predictor. Product rectangle rule is used in Eq. (A.11) to evaluate term

$$\begin{aligned} x_h^P(t_{n+1}) &= g(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, x_h(t_j) x_h(t_j - \tau)) \\ &= g(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, x_h(t_j), x_h(t_{j-k})), \end{aligned} \tag{A.16}$$

where $b_{j,n+1}$ is given as follows

$$b_{j,n+1} = \frac{h^{\alpha}}{\alpha}((n+1-j)^{\alpha} - (n-j)^{\alpha}). \tag{A.17}$$

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Résumé etendu: Ce travail concerne la commande des systèmes multi-agents d'ordre fractionnaires. Les problèmes de consensus et de commande en formation sont étudiés pour la coordination distribuée d'un système multi agents utilisant une topologie de communication directe et fixe, avec et sans retard. Le contenue de la thèse et es principales contributions sont résumées ci dessous.

Dans le chapitre 1, sont présentés : un état de l'art et les outils et définitions du consensus, de la commande en formation, des systèmes multi-agents d'ordre fractionnaire, des topologies de communications et du retard de communication.

Dans le chapitre 2 le problème de la production en formation avec amortissement absolu et retard de communication de systèmes multi-agents d'ordre fractionnaire est étudié. Le cas des systèmes multi-agent d'ordre fractionnaires non linéaires est considéré. Ces systèmes sont réécrits sous forme de systèmes linéaires et un algorithme de commande est proposé en utilisant la théorie de la matrice, la théorie des graphes et l'analyse fréquentielle. Il a été montré que dans le cas des les systèmes dynamiques d'ordre fractionnaire, le choix des fonctions de Lyapunov est plus difficile que dans le cas des systèmes d'ordre entier. Cela nous a conduit à utiliser une méthode d'analyse fréquentielle pour l'analyse de la stabilité des points d'équilibre. Enfin, les résultats de simulation sont respectivement prévus pour valider de notre analyse théorique comparant avec des oeuvres existantes énumérées dans la littérature.

Le chapitre 3 traite du problème de la formation de systèmes multi-agents d'ordre fractionnaires avec amortissement relatif et retard. Dans le chapitre 2, chaque agent atteint la formation finale avec une vitesse nulle (rendez vous statique), alors que

dans certains cas, il est souhaitable que tous les agents atteignent la formation souhaitée et continuent de se déplacer en groupe, au lieu d'un rendez-vous à un point fixe. Dans ce cas, seules des mesures relatives (de position ou de véhicules) sont nécessaires. Tout d'abord, une loi de commande distribuée pour la formation de systèmes multi-agents d'ordre fractionnaire pour des graphes directs et fixes avec amortissement relatif et retard est donnée. Deuxièmement, les conditions de stabilité pour la réalisation de formation avec amortissement relatif et délai de communication sont données en utilisant la méthode d'analyse dans le domaine fréquentiel. Enfin, pour illustrer la validité des résultats obtenus, plusieurs simulations sont présentées sur la base de la méthode prédicteur-correcteur. Une comparaison avec les travaux existants dans la littérature montre l'intérêt de l'approche proposée dans le cas de systèmes d'ordre non entier.

Dans les deux chapitres précédents le cas de la réalisation de formation de systèmes multi-agents sans référence a été traité, alors que dans de nombreuses applications il est souhaitable que les états de tous les agents puissent suivre asymptotiquement un état de référence. C'est pour cette raison que ce chapitre traite du problème du consensus et du suivi de formation de systèmes multi-agents d'ordre fractionnaire basés sur l'erreur de prédiction. Tout d'abord, une loi de commande commune est proposée, et validée par un théorème. Deuxièmement, une loi de commande basée sur l'erreur de prédiction est proposée, et sa validité est également vérifiée par un théorème. La vitesse de convergence de systèmes multi-agents d'ordre fractionnaire avec les deux lois de commande est ensuite comparée. Il a été prouvé que la convergence du système est plus rapide en utilisant la loi basée sur la prédiction d'erreur plutôt que celle de commande commune. Ces lois de commande ont été étendues au cas de la poursuite en formation. Les résultats comparatifs montrent l'intérêt de l'approche proposée dans le cas de systèmes d'ordre non entier. Elles montrent que la vitesse de convergence de systèmes multi-agents d'ordre fractionnaire est plus rapide avec loi de commande basée sur l'erreur de prédiction.

Dans le chapitre 4, les problèmes du consensus et du suivi de formation de systèmes multi-agents d'ordre fractionnaire avec un état de référence ont été étudiés en considérant que tous les agents avaient accès à l'état de référence. Dans ce chapitre 5, nous étudierons les mêmes problèmes mais en considérant que seule une partie des agents a accès cet état de référence. Dans un premier temps, nous avons proposé une loi de commande pour résoudre le problème du consensus de systèmes multi-agents d'ordre fractionnaire avec un état de référence constante. Ensuite, nous avons montré que cette loi de commande ne peut pas garantir un consensus avec un état de référence variant dans le temps. Une novelle loi de commande est alors proposée pour résoudre ce problème, puis étendue pour résoudre le problème u suivi de formation. Enfin, plusieurs simulations sont présentées pour vérifier la validité des résultats obtenus. La comparaison avec les travaux existants montre l'intérêt de notre approche.

La conclusion générale reprend les résultats principaux de la thèse et présente quelques perspectives intéressantes à ce travail visant à résoudre les limitations évoquées ou à étendre les approches proposées, notamment aux cas de systèmes avec un grand nombre d'agents, des commandes présentant des dérivées d'état, de l'extension aux graphes directs commutés, de la prise en compte des perturbations, des multiréférences et de la validation sur une plateforme réelle.

Commande des Systèmes Multi-agent d'Ordre Fractionnaire

Résumé: Ce travail concerne la commande des systèmes multi-agent d'ordre fractionnaire utilisant une topologie de communication fixe. Premièrement, la production en formation avec atténuation absolue et retard de communication est étudiée. Pour cela, une loi de commande et des conditions suffisantes sont proposées. Toutefois, dans certains scénarios, il est souhaitable que tous les agents atteignent la formation souhaitée tout en se déplacent en groupe, au lieu d'un rendez-vous à un point fixe. Ce cas sera traité en étudiant la production en formation avec atténuation relative et retard de communication. Troisièmement, la poursuite par consensus des systèmes multi-agent d'ordre fractionnaire avec un état de référence variable dans le temps est étudiée. Une loi de commande commune et une seconde basée sur la prédiction d'erreur sont proposées, et il a été démontré que le problème du consensus est résolu quand le graphe de communication contient un arbre dirigé. Il a été prouvé que la convergence du système est plus rapide en utilisant la loi de commande commune plutôt que celle basée sur la prédiction d'erreur. Enfin, les lois de commande ci-dessus sont étendues au cas de la poursuite en formation. En effet, dans de nombreux cas, l'information peut être envoyée à partir d'un état de référence vers les agents voisins uniquement et non pas à l'ensemble des agents. Afin de résoudre ce problème, une loi de commande est proposée afin de résoudre le problème du consensus avec un état de référence constant. Puis, deux lois de commande sont proposées afin de résoudre le problème du consensus avec un état de référence variant dans le temps. Ces lois sont étendues pour résoudre le problème de la poursuite en formation.

Mots-Clefs: Commande, Systèms multi-agent, Ordre fractionnaire, Consensus, Production poursuite, Retard de la communication, Atténuation absolue/relative.

Distributed Coordination of fractional-order multi-agent systems

Abstract: This thesis focuses on the distributed coordination of fractional-order multi-agent systems under fixed directed communication graph. Firstly, formation producing with absolute damping and communication delay of fractional-order multi-agent systems is studied. A control law is proposed and some sufficient conditions are derived for achieving formation producing. However, in some scenarios, it might be desirable that all agents achieve formation and move as a group, instead of rendezvous at a stationary point. Therefore, secondly, formation producing with relative damping and communication delay is considered. Thirdly, consensus tracking of fractional-order multi-agent systems with a time-varying reference state is studied. A common control law and a control law based on error predictor are proposed, and it is shown that the control laws are effective when a communication graph has directed spanning trees. Meanwhile, it is proved that the convergence of systems is faster using the control law based on error predictor than by the common one. Finally, the above control laws are extended to achieve formation-tracking problems. In fact, in many cases information can be sent from a reference state to only its neighbor agents not to all the agents. In order to solve the above problem, an effective control law is given to achieve consensus with a constant reference state. Then, an effective general control law and an effective particular one are proposed to achieve consensus with a time-varying reference state. Furthermore, the above control laws are extended to achieve the formation tracking problems.

Keywords: Distributed coordination, Multi-agent systems, Fractional-order, Consensus/Formation producing, Consensus/Formation tracking, Communication delay, Absolute/relative damping.